Evidence of effective axial U(1) symmetry restoration at high temperature QCD

Akio Tomiya (CCNU)

G. Cossu, S. Aoki,

H. Fukaya, T. Kaneko, J. Noaki for JLQCD collaboration

Based on: arXiv:1612.01908 (now submitting to PRD)
PRD 93, no. 3, 034507 (2016)
and related proceedings

(This is not related to this talk but...)

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My paper about Kibble-Zurek physics (in 1+1 dim.) will be available on the arXiv tonight...

Quantum Quench and Scaling of Entanglement Entropy

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[4] *Key Laboratory of Quark & Lepton Physics (MOE) and Institute of Particle Physics, Central China Normal University, Wuhan 430079, CHINA*

Global quantum quench with a finite rate which crosses critical points is known to lead to universal scaling of correlation functions as functions of the quench rate. We explore scaling properties of the entanglement entropy of a subsystem in a harmonic chain during a mass quench which asymptotes to finite constant values at early and late times and for which the dynamics is exactly solvable. Both for fast and slow quenches we find that the entanglement entropy has a constant term plus a term proportional to the subsystem size. For slow quenches, the constant piece is consistent with Kibble-Zurek predictions. Furthermore, the quench rate dependence of the extensive piece enters solely through the instantaneous correlation length at the Kibble-Zurek time, suggesting a scaling hypothesis similar to that for correlation functions.

Cf. Deep inelastic scattering as a probe of entanglement
Dmitri E. Kharzeev, and Eugene M. Levin (arXiv 1702.03489)

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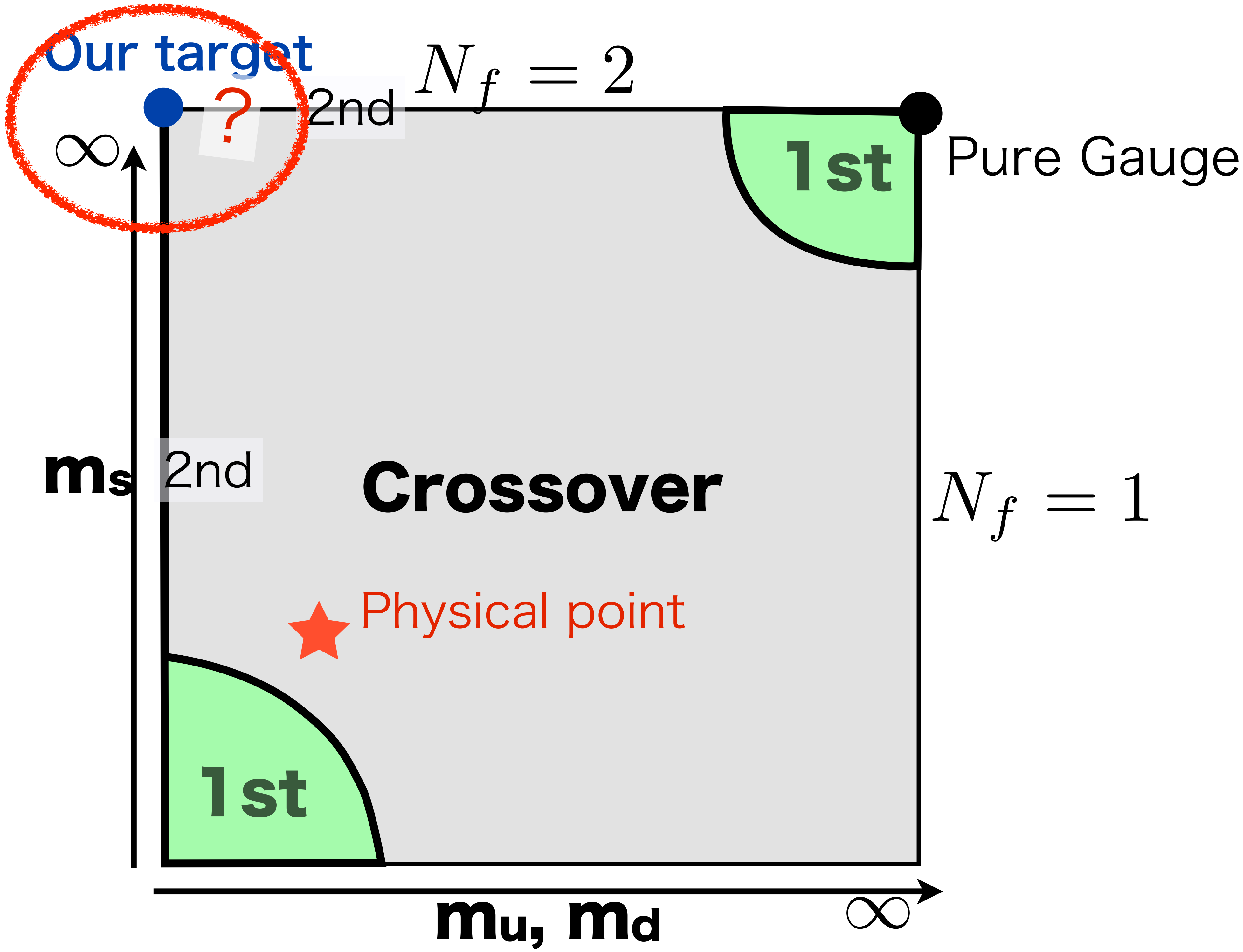
QCD phase transition for various mass?

What happens when Nf=2 at massless limit?



Not directly related to the real physics but useful for model building

Our Question:

Does the **massless** two flavor QCD have $U(1)_A$ symmetry above Tc?

Tool : Lattice QCD

Our Conclusion:

The **massless** two flavor QCD has $U(1)_A$ symmetry above Tc,
if the action has **EXACT** chiral symmetry.

Key word: Chiral symmetry on the lattice

Contents

1. Introduction: $U(1)_A$ sym. in QCD
2. Our observables: Dirac spectrum
3. overlap & domain-wall fermion
4. Setup & Results
5. “Ginsparg-Wilson violation” for Domain-wall fermion in low-laying modes
6. Summary

1. Introduction for U(1)A sym. in QCD

SU(2) chiral symmetry is broken spontaneously, U(1) is by the anomaly

$T = 0$

QCD Lagrangian

$$\underbrace{SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}}}_{\text{SSB}} \times U(1)_{\mathrm{V}} \times \underbrace{U(1)_{\mathrm{A}}}_{\textbf{Anomaly}}$$

$$\longrightarrow SU(2)_{\mathrm{V}} \times U(1)_{\mathrm{V}}$$: Symmetry of theory

What is the anomaly?

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What is the anomaly? $\psi = {}^{\top}(u \quad d)$

$S = \int d^4x \overline{\psi} \not{D} \psi$ is invariant under $\left\{ \begin{array}{l} \psi \to e^{i\theta\gamma_5}\psi \\ \overline{\psi} \to \overline{\psi} e^{i\theta\gamma_5} \end{array} \right.$ Namely sym.

Because : $\gamma_5 \not{D} + \not{D}\gamma_5 = 0$

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Because : $\gamma_5 \not{D} + \not{D}\gamma_5 = 0$

but **the path integral measure is not invariant!** **non-trivial Jacobian.**

$$\mathcal{D}\overline{\psi}\mathcal{D}\psi \rightarrow \mathcal{D}\overline{\psi}\mathcal{D}\psi e^{i\Gamma}$$ Anomaly(Fujikawa 1972)

This effect must exist for explanation of heavy η'

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On the other hand,

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On the other hand,

$T > T_c$

$$SU(2)_{\mathrm{V}} \longrightarrow SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}}$$ Restored

$$U(1)_{\mathrm{A}} \longrightarrow \textbf{??}$$

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$$U(1)_{\mathrm{A}} \longrightarrow \textbf{??}$$

What happens to the anomaly above Tc?

1. Introduction for U(1)A sym. in QCD

Symmetry leads degeneracy between mesons

$$
\begin{tikzcd}
\langle \pi(x)\pi(0) \rangle \arrow[r, leftrightarrow, "SU(2)_L \times SU(2)_R"] \arrow[d, leftrightarrow, "U(1)_A"'] & \langle \sigma(x)\sigma(0) \rangle \arrow[d, leftrightarrow, "U(1)_A"] \\
\langle \delta(x)\delta(0) \rangle \arrow[r, leftrightarrow, "SU(2)_L \times SU(2)_R"] & \langle \eta(x)\eta(0) \rangle
\end{tikzcd}
$$

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\end{tikzcd}
$$

$$\chi_{U(1)_A} \equiv \int d^4x [\underline{\langle \pi(x)\pi(0) \rangle} - \underline{\langle \delta(x)\delta(0) \rangle}]$$

"Order parameter" of U(1)A

If this quantity(susceptibility) is 0 at V →∞, m→0,
U(1)A symmetry is <u>effectively</u> "restored"
(in other wards, invisible)

2. Our observables: Dirac spectrum

$\rho(\lambda)$ is a spectrum of the Dirac operator with QCD background

Our observable

$$(\gamma_5 \underline{D\!\!\!/})\psi_j = \lambda_j \psi_j$$

(Covariant derivative has information of the gauge field)

Eigenvalue equation can be solved
for a given gauge configuration

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-> λ s are distributed in a certain way,
= the Dirac spectrum $\underline{\rho(\lambda)}$

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Eigenvalue equation can be solved for a given gauge configuration

One can repeat for all configurations
-> λ s are distributed in a certain way,
= the Dirac spectrum $\underline{\rho(\lambda)}$

The Dirac spectrum $\underline{\rho(\lambda)}$ has information of <u>symmetry of quarks</u>

2. Our observables: Dirac spectrum

If ρ has a (volume insensitive) gap, U(1) is effectively restored

For SU(2): The Banks-Casher relation

$$\langle\bar{\psi}\psi\rangle = \int_0^\infty d\lambda\, \rho(\lambda) \frac{2m}{\lambda^2 + m^2} \qquad \rho(\lambda) = \lim_{V\to\infty} \frac{1}{V} \sum_n \langle \delta(\lambda_n^A - \lambda) \rangle_A$$

$$|\langle\bar{\psi}\psi\rangle| = \pi\rho(0) = 0 \longrightarrow \text{SU(2) restoration}$$

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For U(1): Cohen's argument $\chi_{U(1)_A} \equiv \int d^4x [\langle \pi(x)\pi(0)\rangle - \langle \delta(x)\delta(0)\rangle]$

$$\chi_{U(1)_A} = \int_0^\infty d\lambda\, \rho(\lambda) \frac{4m^2}{(\lambda^2 + m^2)^2}$$

$$\text{IF } \rho(\lambda < \lambda_{\mathrm{cr}}) = 0 \longrightarrow \chi_{U(1)_A} = 0$$

SU(2) and U(1) restoration

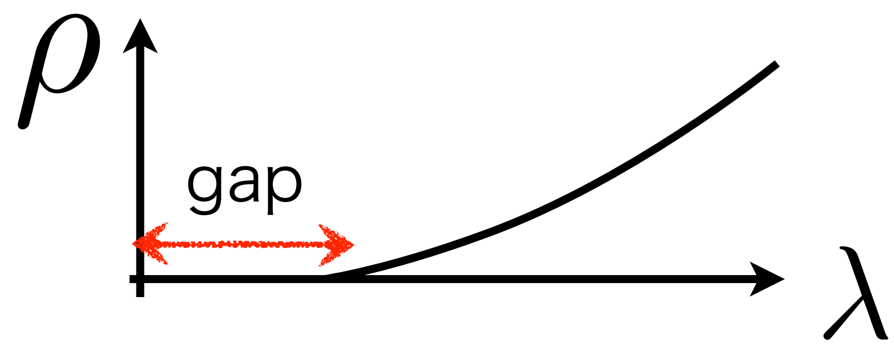
Cohen(1996), Aoki-Fukaya-Taniguchi (2012)

2. Our observables: Dirac spectrum

If ρ has a (volume insensitive) gap, U(1) is effectively restored

Argument by Cohen(1996)

If there is a gap in the Dirac spectrum



"U(1)$_A$ violation"

$$\int d^4x[\langle\pi(x)\pi(0)\rangle - \langle\delta(x)\delta(0)\rangle] = \int_0^\infty d\lambda \frac{4m^2\rho(\lambda)}{(m^2+\lambda^2)^2}$$

$$\rightarrow \underline{0 \ \ (m \rightarrow 0)}$$

invisible

Cf : Aoki-Fukaya-Taniguchi (2012):
λ^3 may be enough for U(1)$_A$ effective restoration.

low-laying modes are essential for this argument!

1. Introduction for $U(1)_A$ sym. in QCD

Symmetry leads degeneracy between mesons

Previous studies (DW type) are controversial !

Group	Fermion	Size	Gap in the spectrum	$U_A(1)$ Correlator	$U(1)_A$ @Tc
JLQCD (2013)	**Overlap (Top. fixed)**	2 fm	**Gap**	Degenerate	**Restored**
TWQCD (2013)	Optimal domain-wall	3 fm	**No gap**	Degenerate	**Restored ?**
LLNL/RBC, Hot QCD (2013, 2014)	**(Mobius)-Domain-wall (W/ ov)**	2, 4, 11 fm	**No gap**	No degeneracy	**Violated**

What makes such difference?

Fermion(Chiral sym.), Volumes or Topology ?

3. Domain-wall and overlap fermion

Chiral symmetry on the lattice = Ginsparg-Wilson relation

SU(2) and U(1) are parts of chiral symmetry in the action:

- Chiral symmetry in continuum theory

$$\gamma_5 \not{D} + \not{D} \gamma_5 = 0$$

3. Domain-wall and overlap fermion

Chiral symmetry on the lattice = Ginsparg-Wilson relation

SU(2) and U(1) are parts of chiral symmetry in the action:

- Chiral symmetry in continuum theory

$$\gamma_5 \not{D} + \not{D}\gamma_5 = 0$$

$$\Downarrow$$

☆ Chiral symmetry on the lattice (Cf. Nielsen-Ninomiya thm)

$$\gamma_5 \not{D} + \not{D}\gamma_5 = 2a\not{D}\gamma_5\not{D}$$

(Here “a” is a lattice spacing)

“Ginsparg-Wilson relation”

3. Domain-wall and overlap fermion

Chiral symmetry on the lattice = Ginsparg-Wilson relation

"Ginsparg-Wilson relation"

$$\gamma_5 \not{D} + \not{D}\gamma_5 = 2a\not{D}\gamma_5\not{D}$$

If D satisfies GW relation…

3. Domain-wall and overlap fermion

Chiral symmetry on the lattice = Ginsparg-Wilson relation

"Ginsparg-Wilson relation"

$$\gamma_5 \not{D} + \not{D}\gamma_5 = 2a\not{D}\gamma_5\not{D}$$

If D satisfies GW relation…

(1) It has "exact" chiral symmetry

$$\psi \to \psi' = e^{i\gamma_5(1-aD)\theta}\psi$$

$$\bar{\psi} \to \bar{\psi}' = \bar{\psi}e^{i\gamma_5\theta}$$

(2) $U(1)_A$ symmetry is broken by the Jacobian as same as the continuum theory

(3) It satisfies the Atiyah-Singer index theorem

3. Domain-wall and overlap fermion

Overlap fermion satisfies the Ginsparg-Wilson relation

The overlap Dirac operator satisfies GW relation

$$D_{\mathrm{ov}} = \frac{1+m}{2} - \frac{1-m}{2}\gamma_5 \mathrm{sgn}(H_T)$$

However…
numerical cost of the sign function is extremely expensive!

There is an approximate one,
“The domain-wall fermion”

3. Domain-wall and overlap fermion

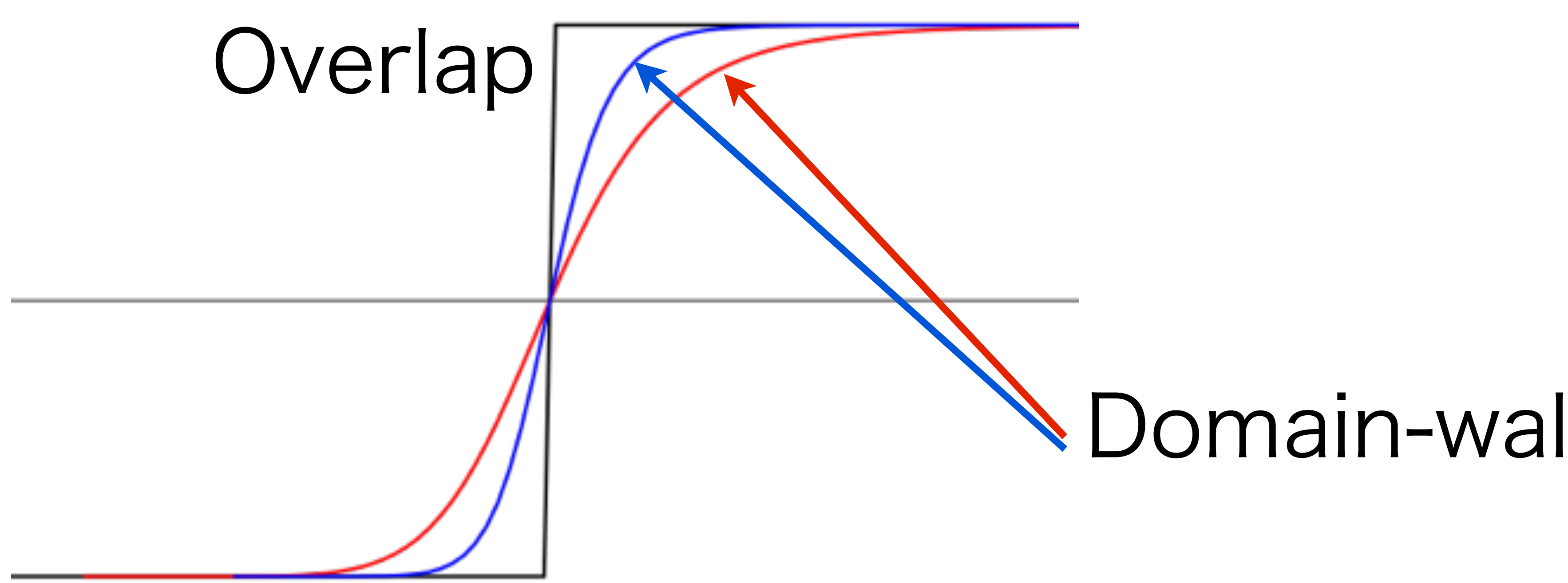
Overlap fermion satisfies the Ginsparg-Wilson relation

Domain-wall fermion ~ Overlap fermion + m_{res}

$$D_{\mathrm{ov}} = \frac{1+m}{2} - \frac{1-m}{2}\gamma_5 \mathrm{sgn}(H_T)$$

approximate

Domain-wall fermion: $\tanh\left[L_s \tanh^{-1}(2H_T)\right]$



Qualitative difference can be measured by "residual mass": m_{res}

4. Setup & Results

Sea quark: Domain-wall and Reweighted Overlap, Probe: DW and OV

Our Setup

1. Sea quarks : Dynamical Möbius domain-wall fermion with **small m_{res}**.
2. Calculation is done with and without OV/DW reweighting to realize overlap sea-quark effectively
3. Volume & topology : 3 Volumes (2-4 fm) and frequent topology tunneling.
4. Probes : Domain-wall and overlap valence quarks
5. Temperature range: 172 MeV to 217 MeV. Tc ~ 190 MeV

4. Setup & Results

Sea quark: Domain-wall(DW) and Reweighted Overlap(OV), Probe: DW and OV

$L^3 \times L_t$	β	ma	L_s	$m_{\rm res}a$	T [MeV]	#trj	$N_{\rm conf}$	$N^{\rm eff}_{\rm conf}$	$N^{\rm eff(2)}_{\rm conf}$	$\tau^{\rm CG}_{\rm int}$	$\tau^{\rm top}_{\rm int}$	$M_{PS}L$
$16^3 \times 8$	4.07	0.01	12	0.00166(15)	203(1)	6600	239	11(13)	45(8)	70	25(6)	5.4(3)
$16^3 \times 8$	4.07	0.001	24	0.00097(43)	203(1)	12000	197	7 (7)	14(3)	315	23(4)	5.3(4)
$16^3 \times 8$	4.10	0.01	12	0.00079(5)	217(1)	7000	203	23(7)	150(17)	134	30(10)	6.9(5)
$16^3 \times 8$	4.10	0.001	24	0.00048(14)	217(1)	12000	214	31(10)	121(10)	104	24(4)	6.3(9)
$32^3 \times 8$	4.07	0.001	24	0.00085(9)	203(1)	4200	210	10(3)*	–	128	18(4)	11.7(9)
$32^3 \times 8$	4.10	0.01	12	0.0009(5)	217(1)	3800	189	9(4)*	–	125	30(10)	12.6(5)
$32^3 \times 8$	4.10	0.005	24	0.00053(4)	217(1)	3100	146	20(4)*	–	84	24(9)	11.6(7)
$32^3 \times 8$	4.10	0.001	24	0.00048(5)	217(1)	7700	229	18(5)*	–	10	23(5)	12.3(9)
$32^3 \times 12$	4.18	0.01	16	0.00022(5)	172(1)	2600	(319)	–	–	–	–	5.8(1)
$32^3 \times 12$	4.20	0.01	16	0.00020(1)	179(1)	3400	(341)	–	–	–	–	–
$32^3 \times 12$	4.22	0.01	16	0.00010(1)	187(1)	7000	(703)	–	–	–	–	5.4(2)
$32^3 \times 12$	4.23	0.01	16	0.00008(1)	191(1)	5600	51	28(4)	38(5)	240	120(50)	–
$32^3 \times 12$	4.23	0.005	16	0.00012(1)	191(1)	10300	206	22(2)	27(2)	131	160(140)	–
$32^3 \times 12$	4.23	0.0025	16	0.00016(4)	191(1)	9400	195	16(2)	255(31)	85	110(30)	–
$32^3 \times 12$	4.24	0.01	16	0.00008(1)	195(1)	7600	49	23(5)	36(5)	125	100(40)	6.8(5)
$32^3 \times 12$	4.24	0.005	16	0.00010(2)	195(1)	9700	190	9(18)	53(6)	84	130(30)	–
$32^3 \times 12$	4.24	0.0025	16	0.00011(2)	195(1)	16000	188	8(10)	7(1)	618	80(20)	6(2)

(1) m_{res} is enough small

(2) # of statistics are increased from 2015

(3) We care about finite size effect & "overlapping problem" for reweighting

Calculations done by BG/Q and SR16000 in KEK using Iroiro++

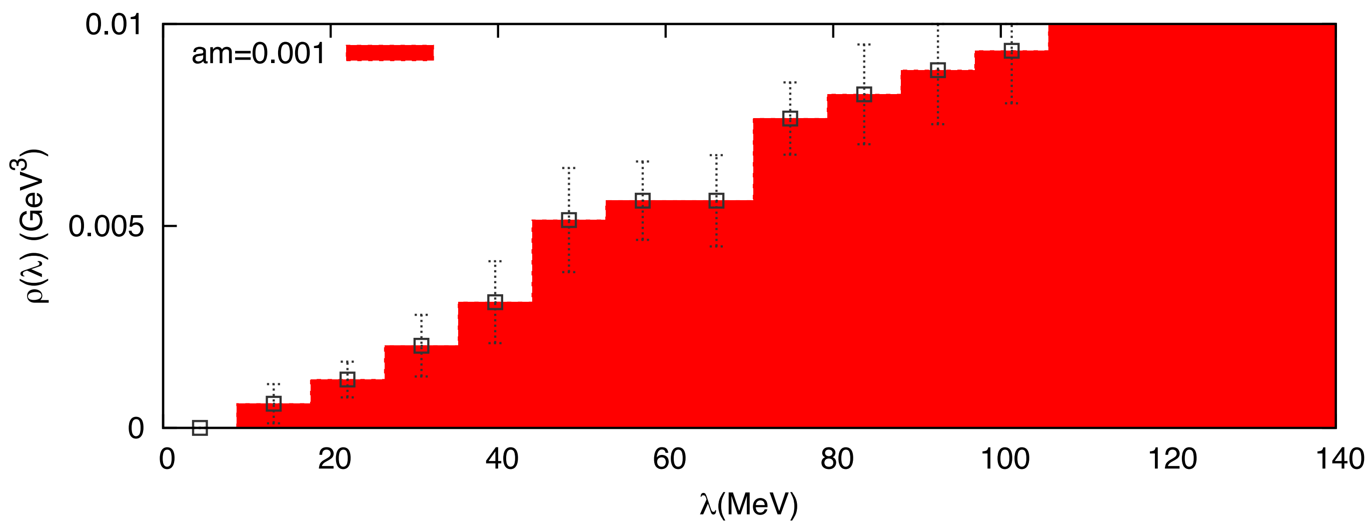
 arXiv: 1612.01908 & PRD 93, no. 3, 034507 (2016)

4. Setup & Results

Reweighted Overlap with overlap probe has gap! and volume insensitive!!

T= 203 MeV for L=2fm, T=1.13 Tc (small lattice)

Domain-wall
on DW sea



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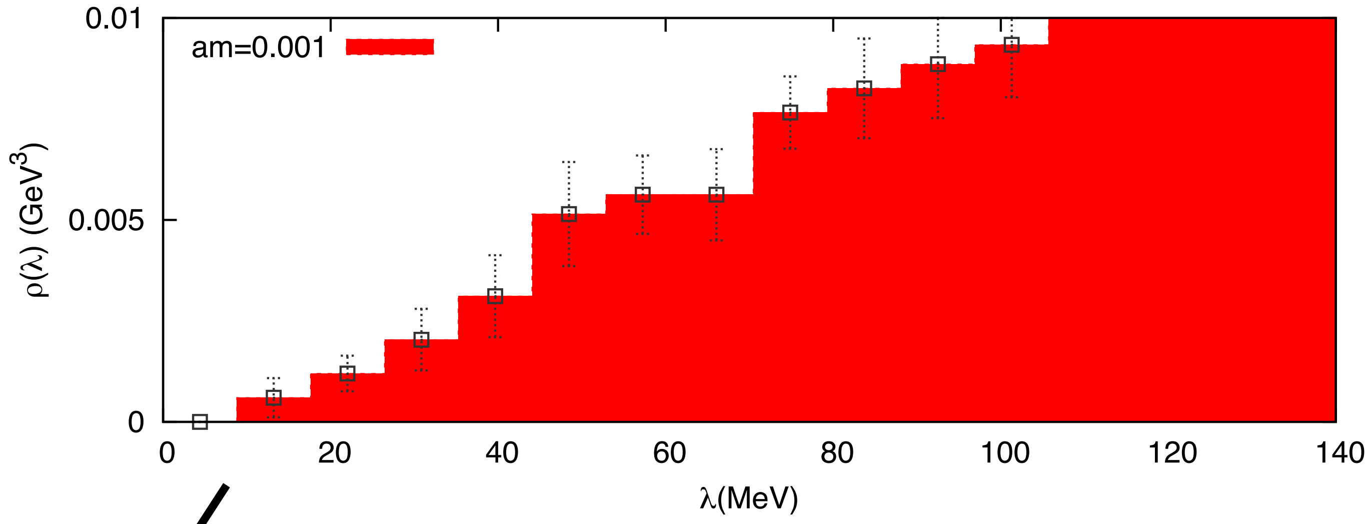
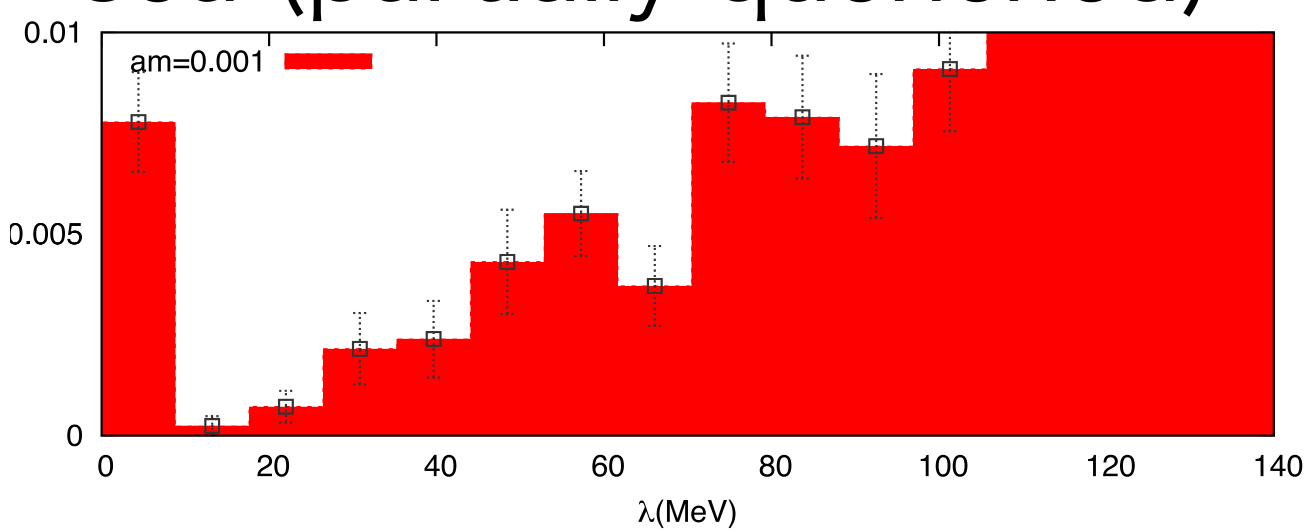


Overlap on domain-wall
sea (partially quenched)



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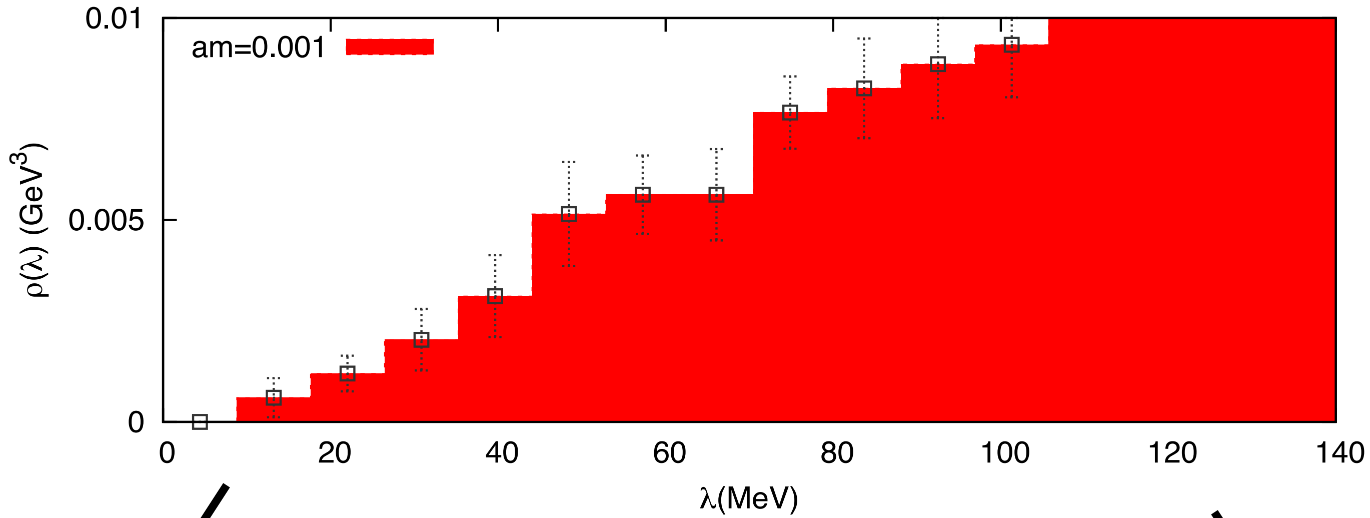
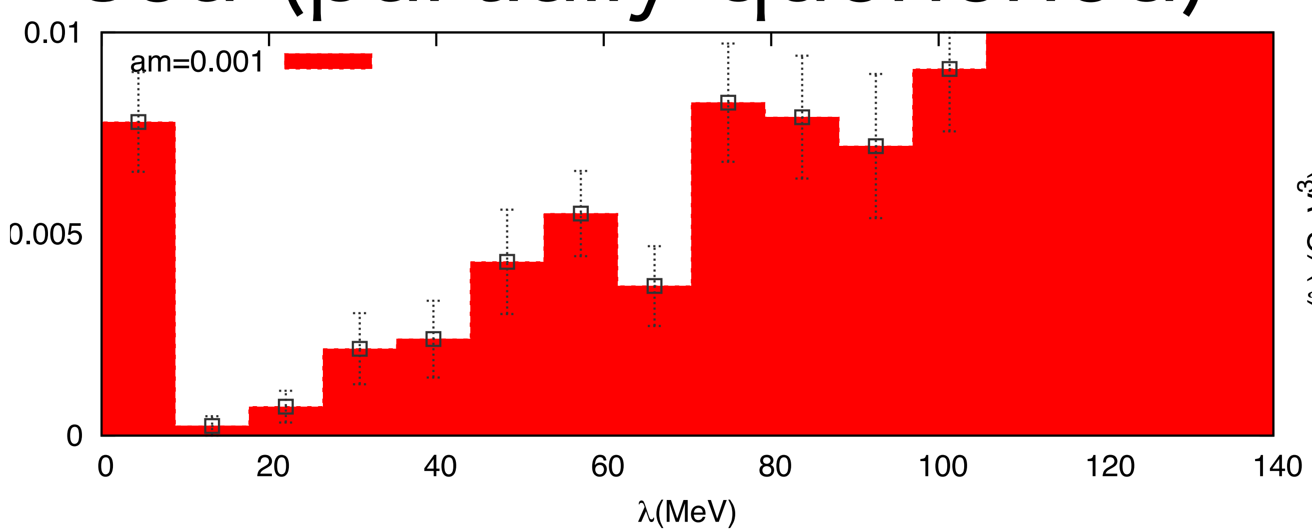
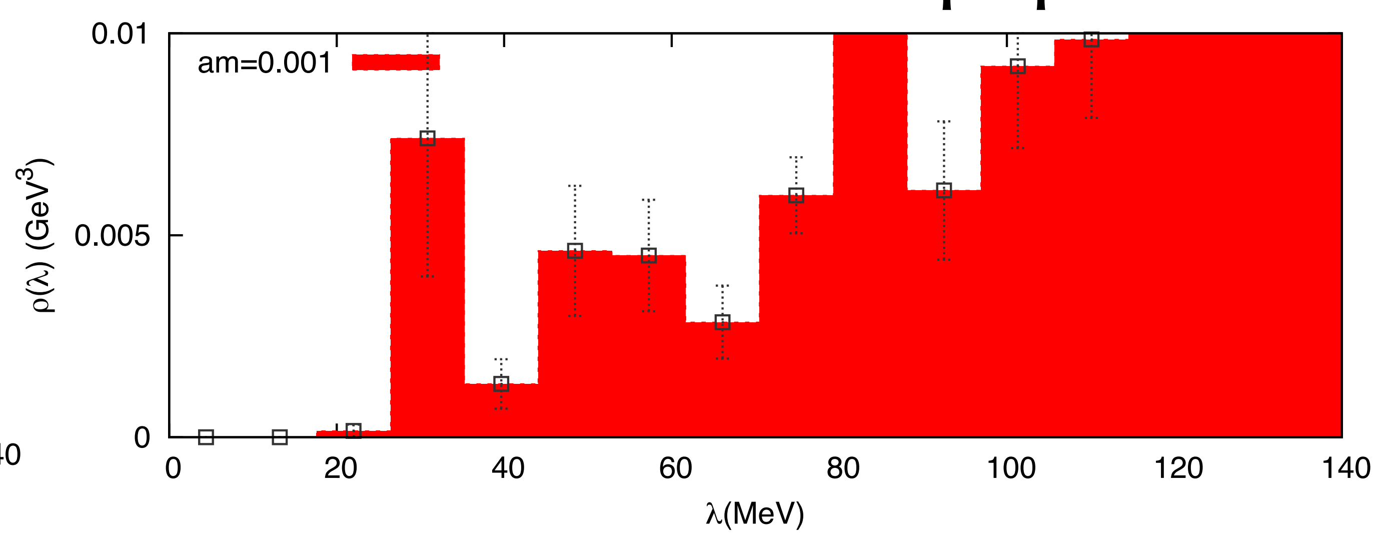


Overlap on domain-wall sea (partially quenched)



(reweighted)Overlap sea with overlap probe



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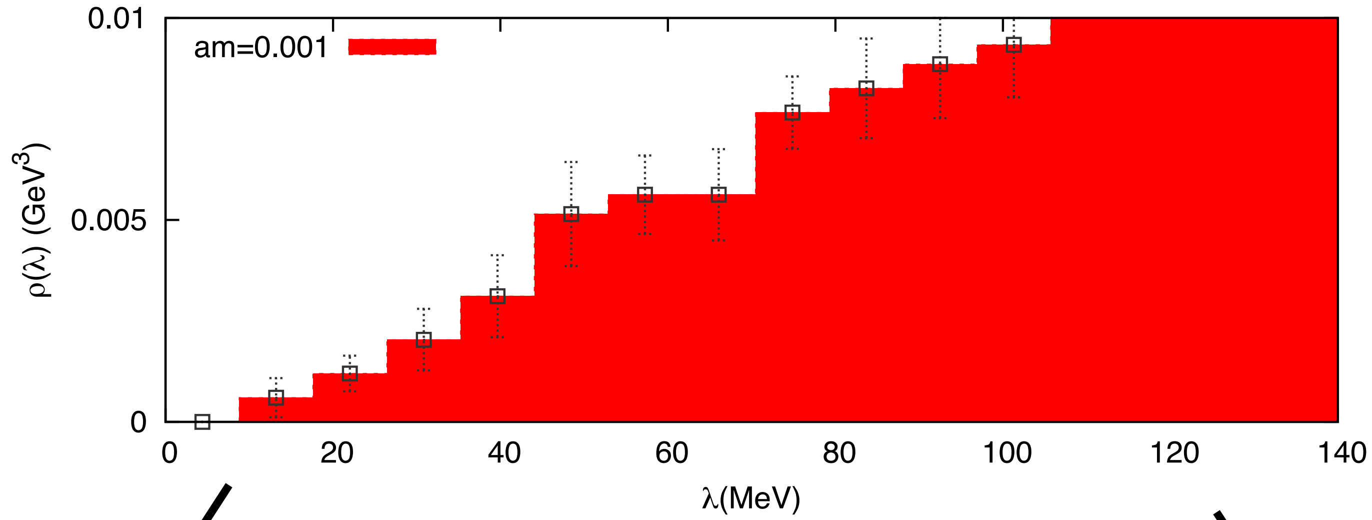
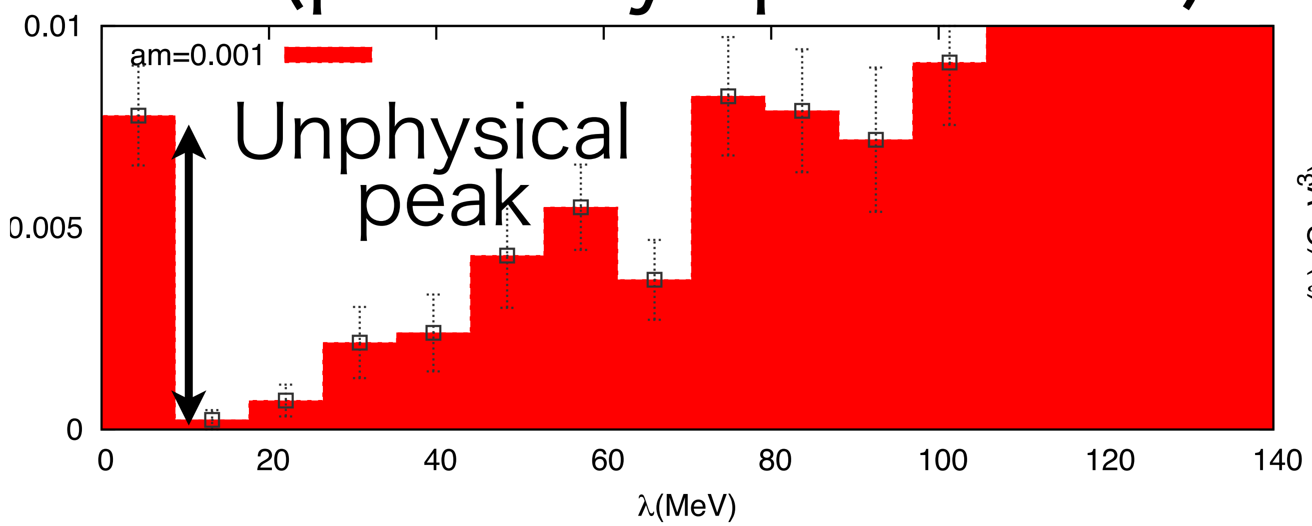
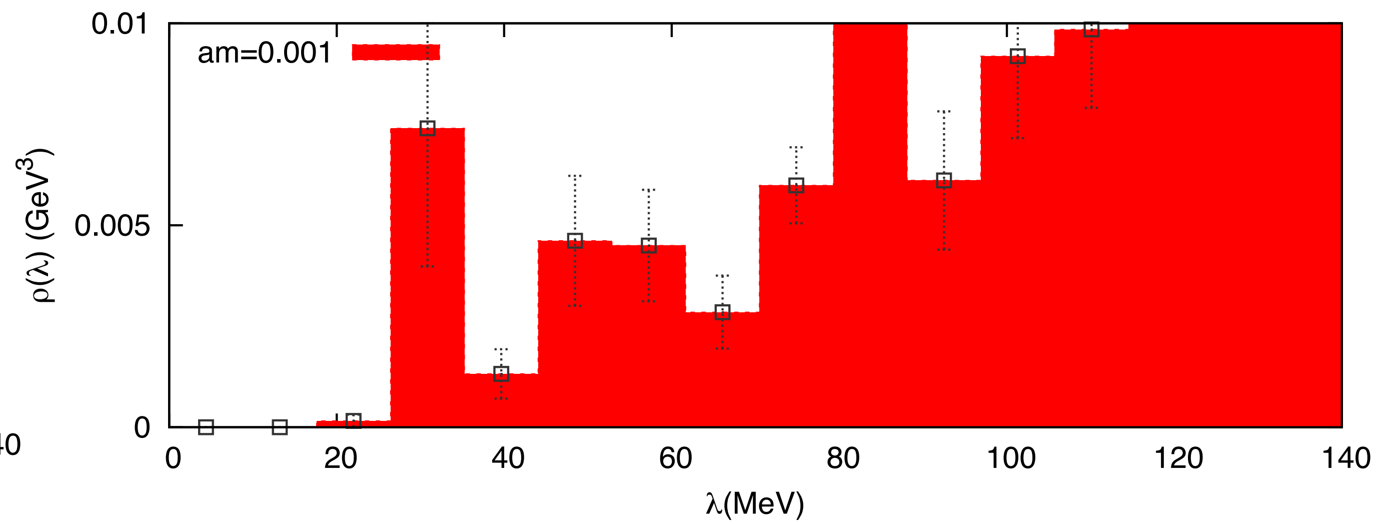
on DW sea



Overlap on domain-wall sea (partially quenched)



(reweighted)Overlap sea with overlap probe

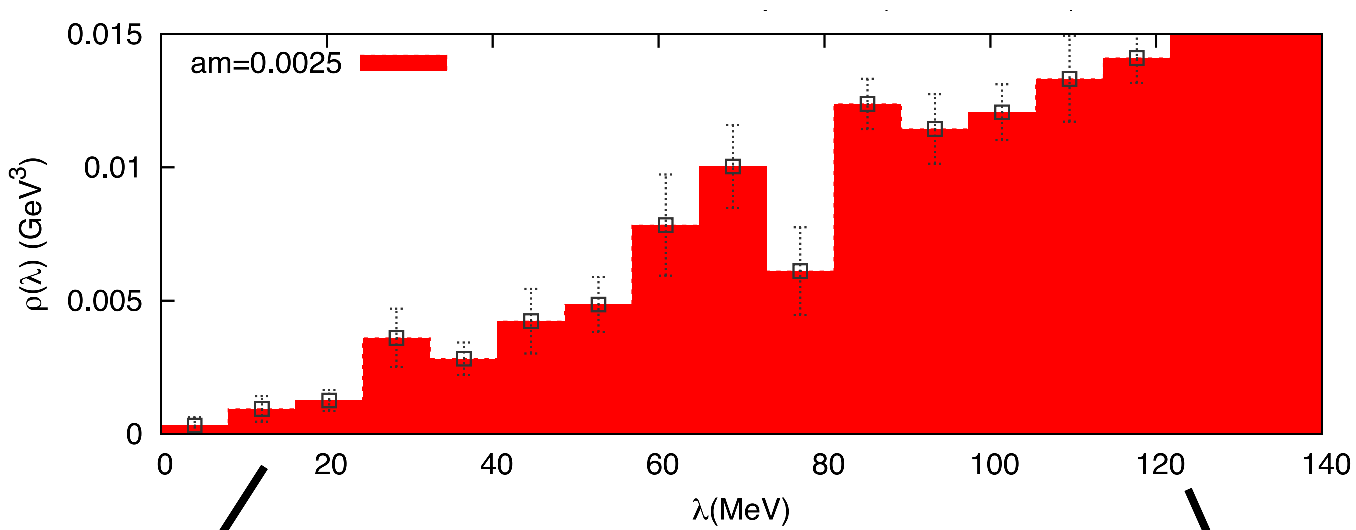


4. Setup & Results

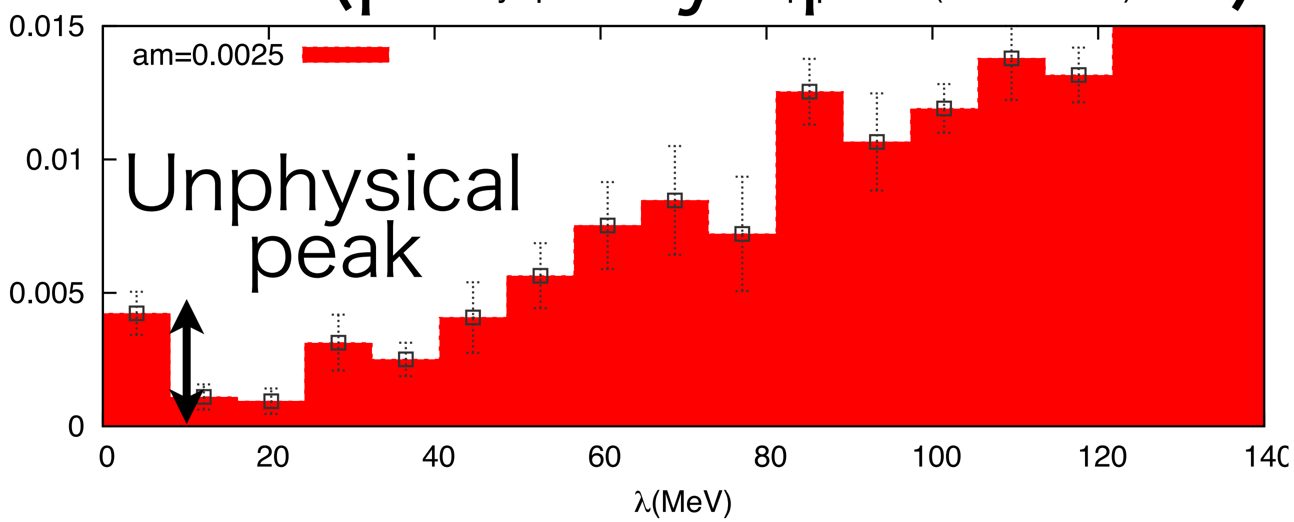
Reweighted Overlap with overlap probe has gap! and volume insensitive!!

T= 190 MeV for L=3fm,T=1.05 Tc (middle size, finer lattice)

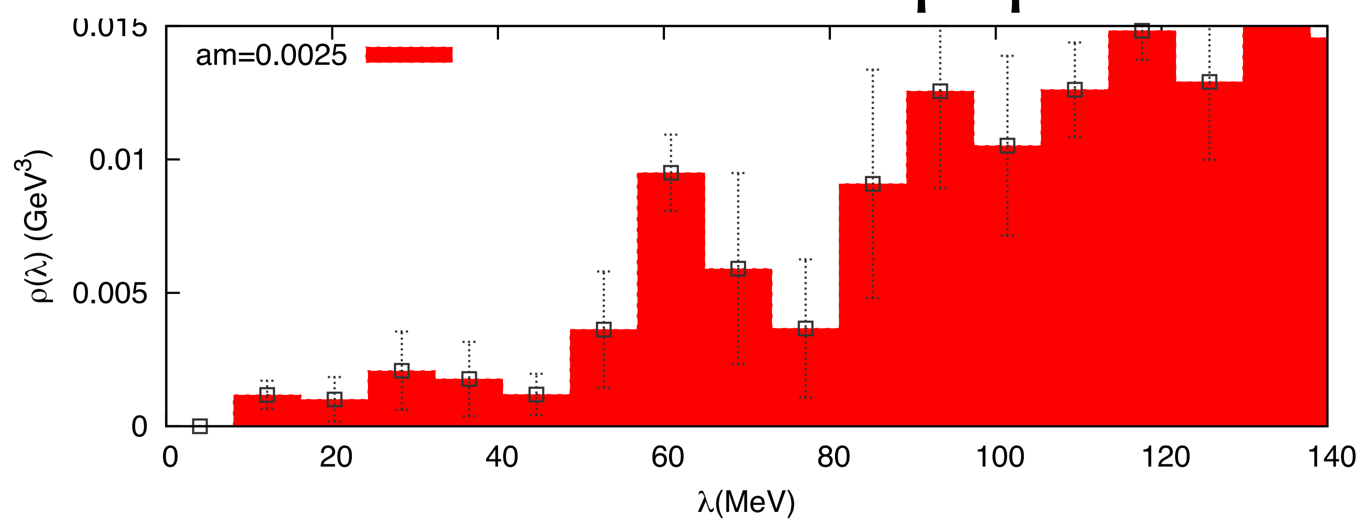
Domain-wall
on DW sea



Overlap on domain-wall sea (partially quenched)



(reweighted)Overlap sea with overlap probe



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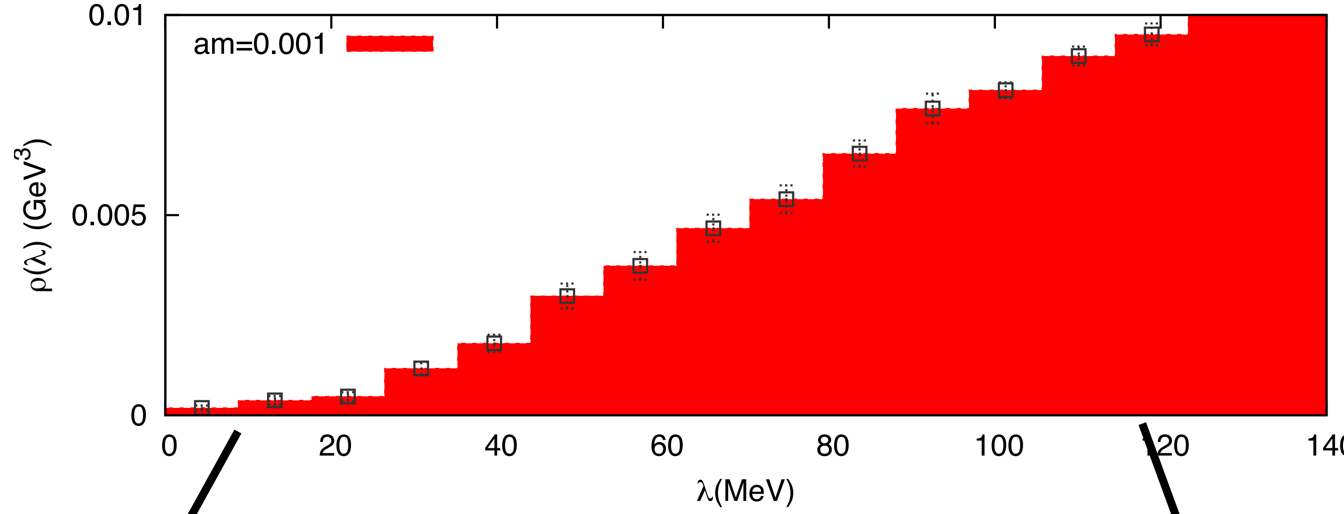
4. Setup & Results

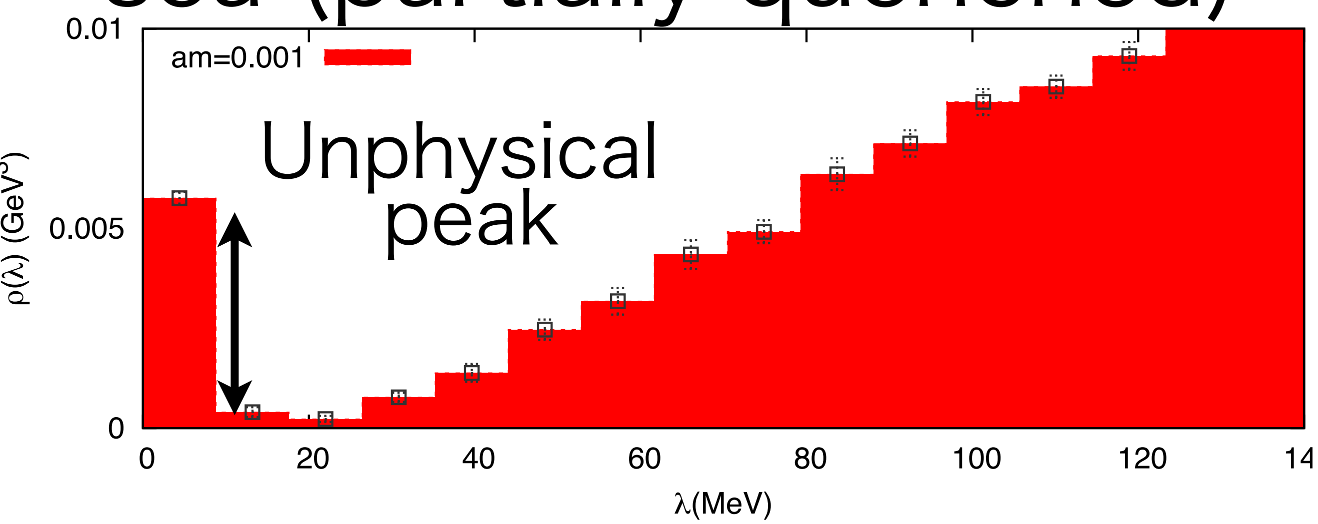
Reweighted Overlap with overlap probe has gap! and volume insensitive!!

T= 202 MeV for L=4fm, T=1.1 Tc (Large volume)

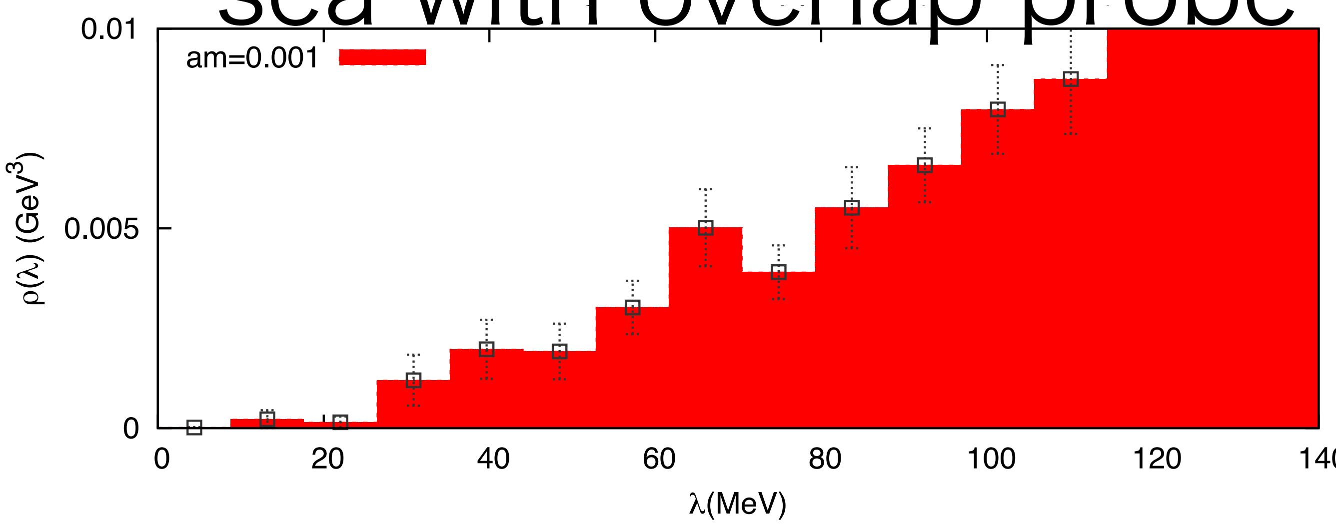
Domain-wall
on DW sea



Overlap on domain-wall sea (partially quenched)



(reweighted)Overlap sea with overlap probe

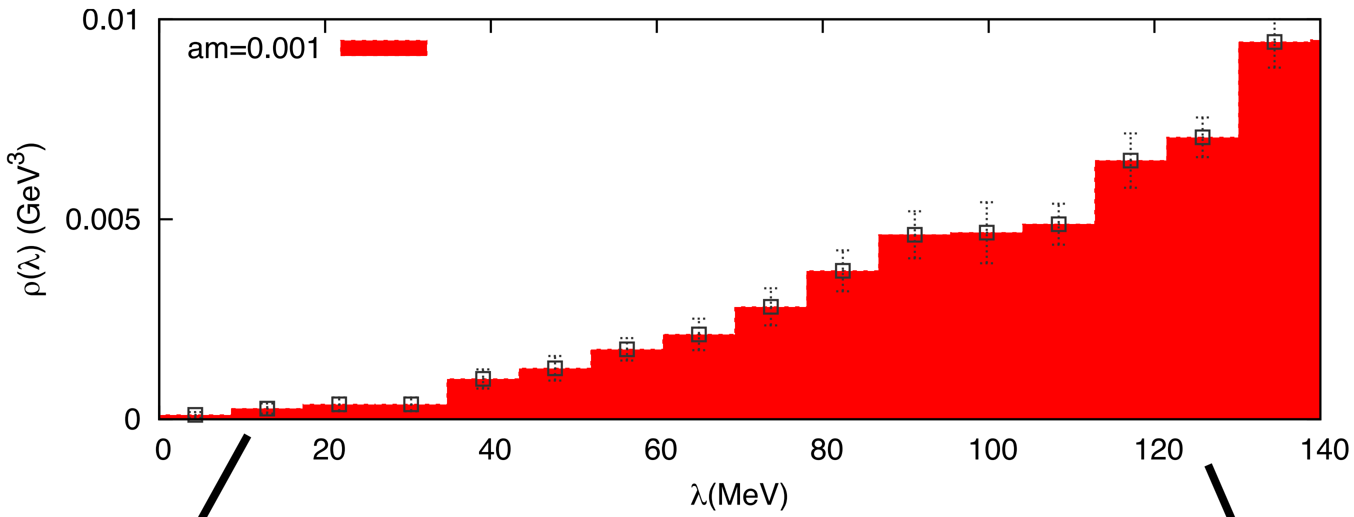


4. Setup & Results

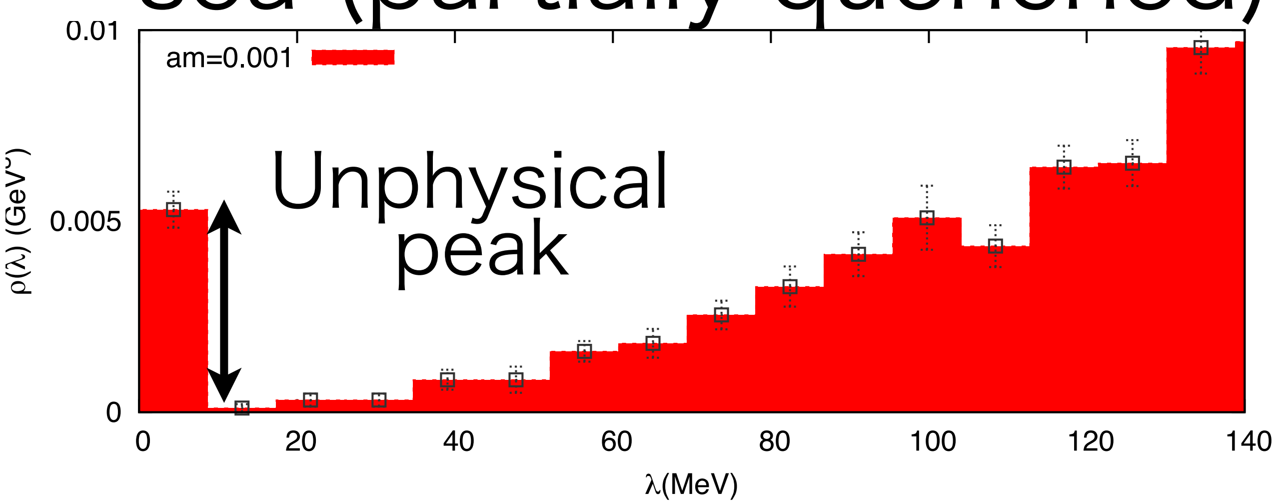
Reweighted Overlap with overlap probe has gap! and volume insensitive!!

T= 217 MeV for L=4fm, T=1.2 Tc (Large volume)

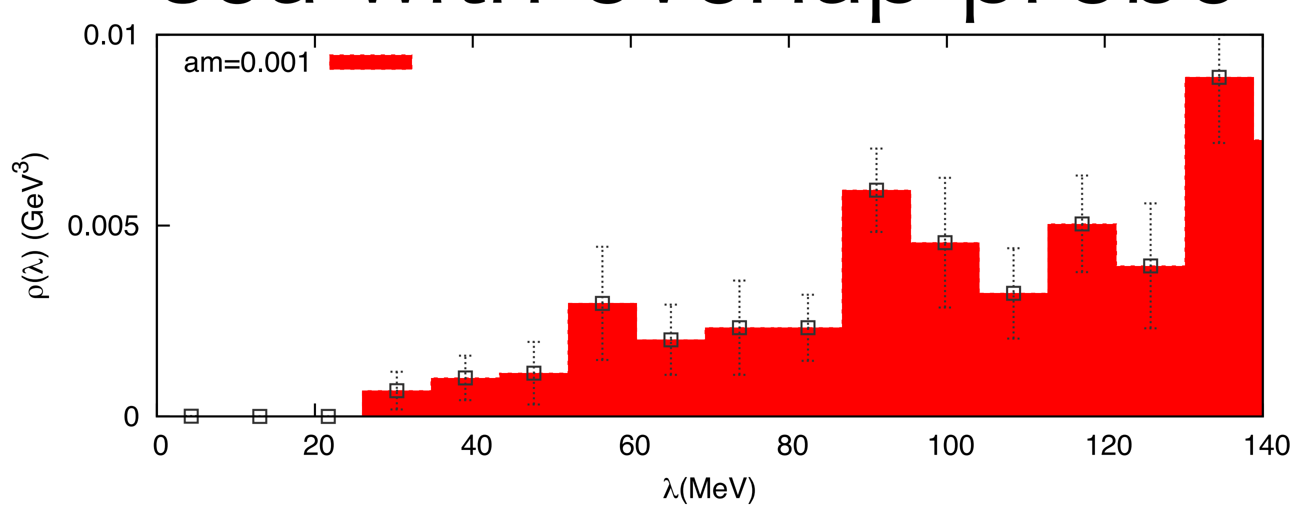
Domain-wall on DW sea



Overlap on domain-wall sea (partially quenched)



(reweighted)Overlap sea with overlap probe



Why they look different??

5. "GW violation" for DW fermion in low-laying modes

Difference coming from violation of Ginsparg-Wilson relation in low-laying modes

To understand difference between spectra, we define Ginsparg-Wilson relation violation for **individual** eigenmode:

$$g_i \propto \psi_i^\dagger \gamma_5 [D\gamma_5 + \gamma_5 D - 2aD\gamma_5 D]\psi_i$$

ψ: Eigenmodes of the Dirac operator D

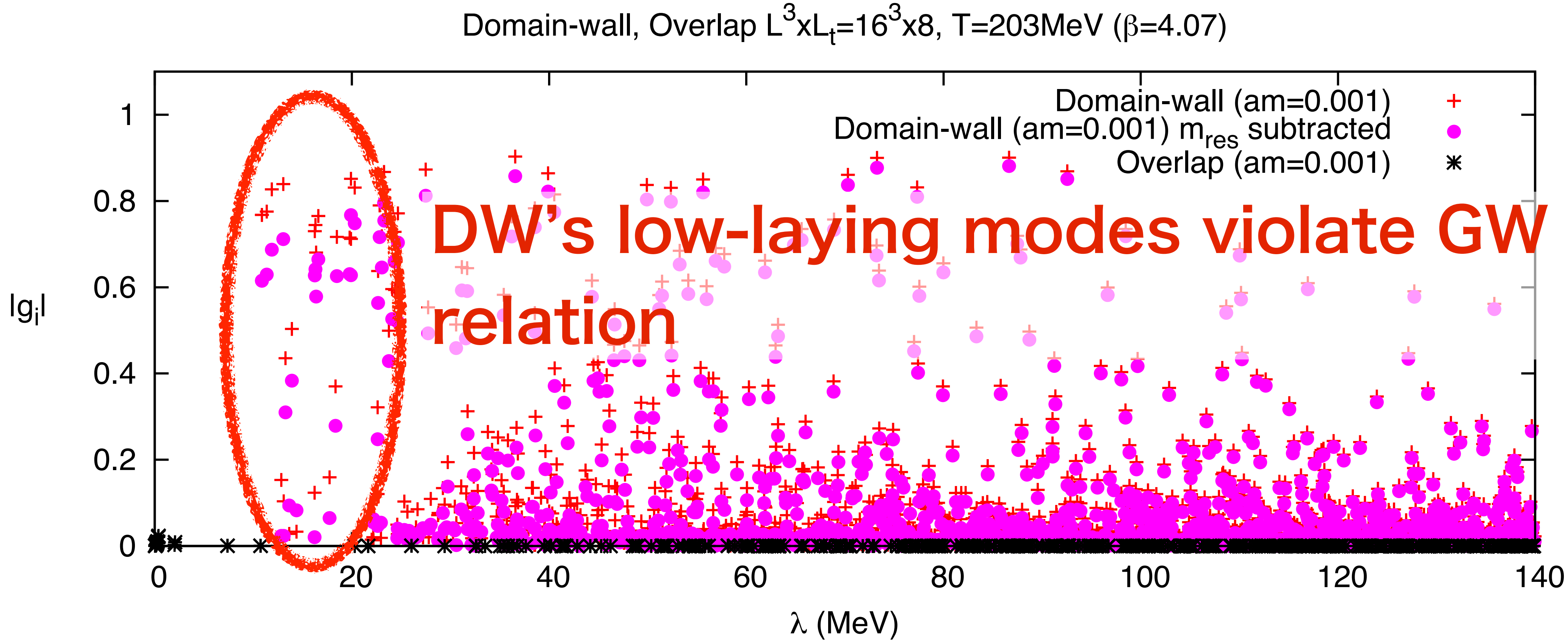
※ This "g" is zero for the overlap Dirac op.

5. “GW violation” for DW fermion in low-laying modes

Difference coming from violation of Ginsparg-Wilson relation in low-laying modes

$$g_i \propto \psi_i^\dagger \gamma_5 [D\gamma_5 + \gamma_5 D - 2aD\gamma_5 D]\psi_i$$

$|g_i|$= |GW violation|



The lattice artifact can be 100 % for the near zero-modes for Domain-wall fermion

 arXiv: 1612.01908 & PRD 93, no. 3, 034507 (2016)

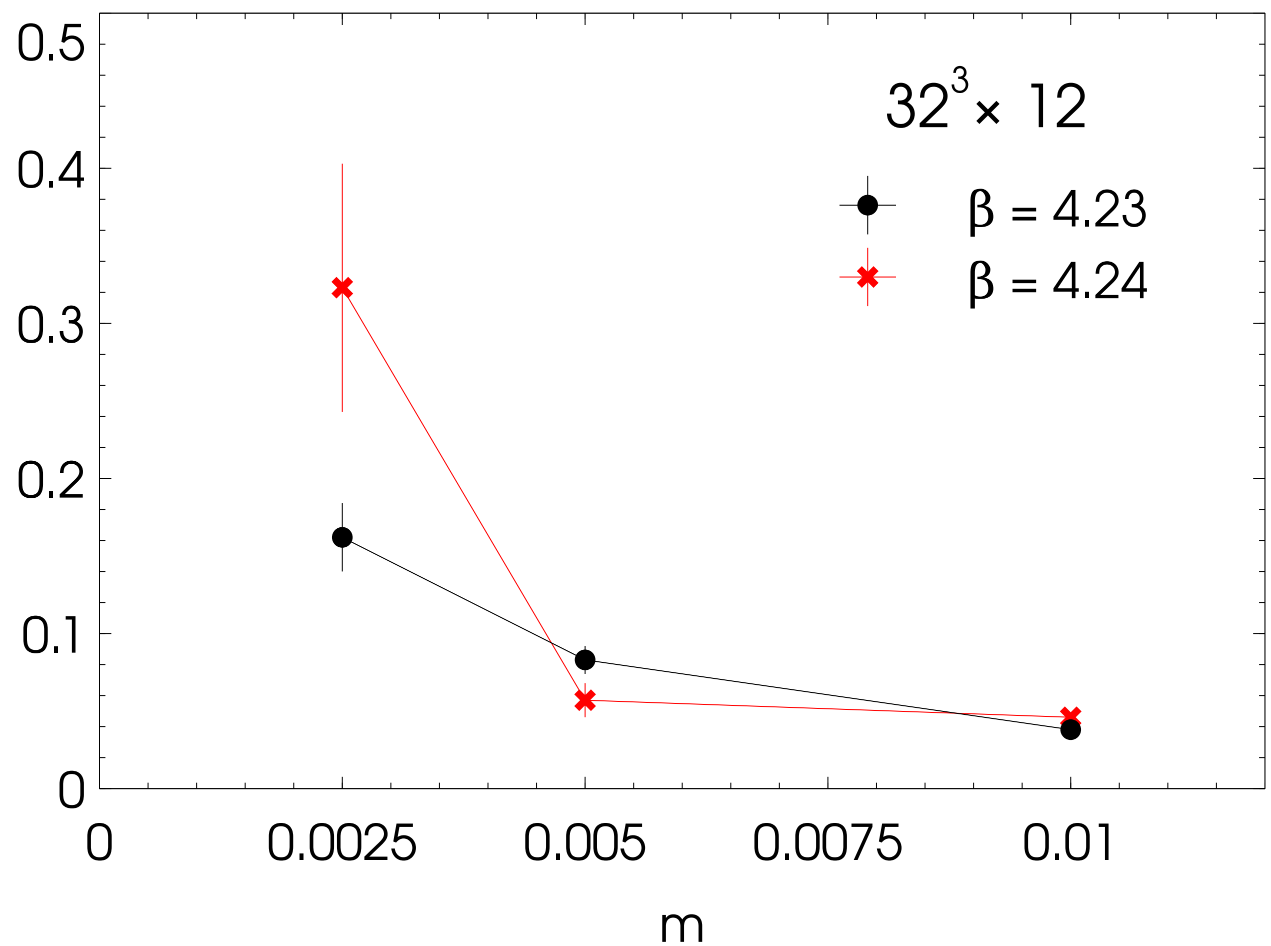
5. “GW violation” for DW fermion in low-laying modes

Susceptibility is dominated by Ginsparg-Wilson violation

$\chi_{\mathrm{U(1)}}$ also has GW violation

$$\chi_{U(1)_A} \equiv \int d^4x [\langle \pi(x)\pi(0)\rangle - \langle \delta(x)\delta(0)\rangle]$$

Ratio of susceptibility:
GW-breaking-modes v.s.
Total
for DW fermion



Finer lattice
1/a ~ 2.2 GeV

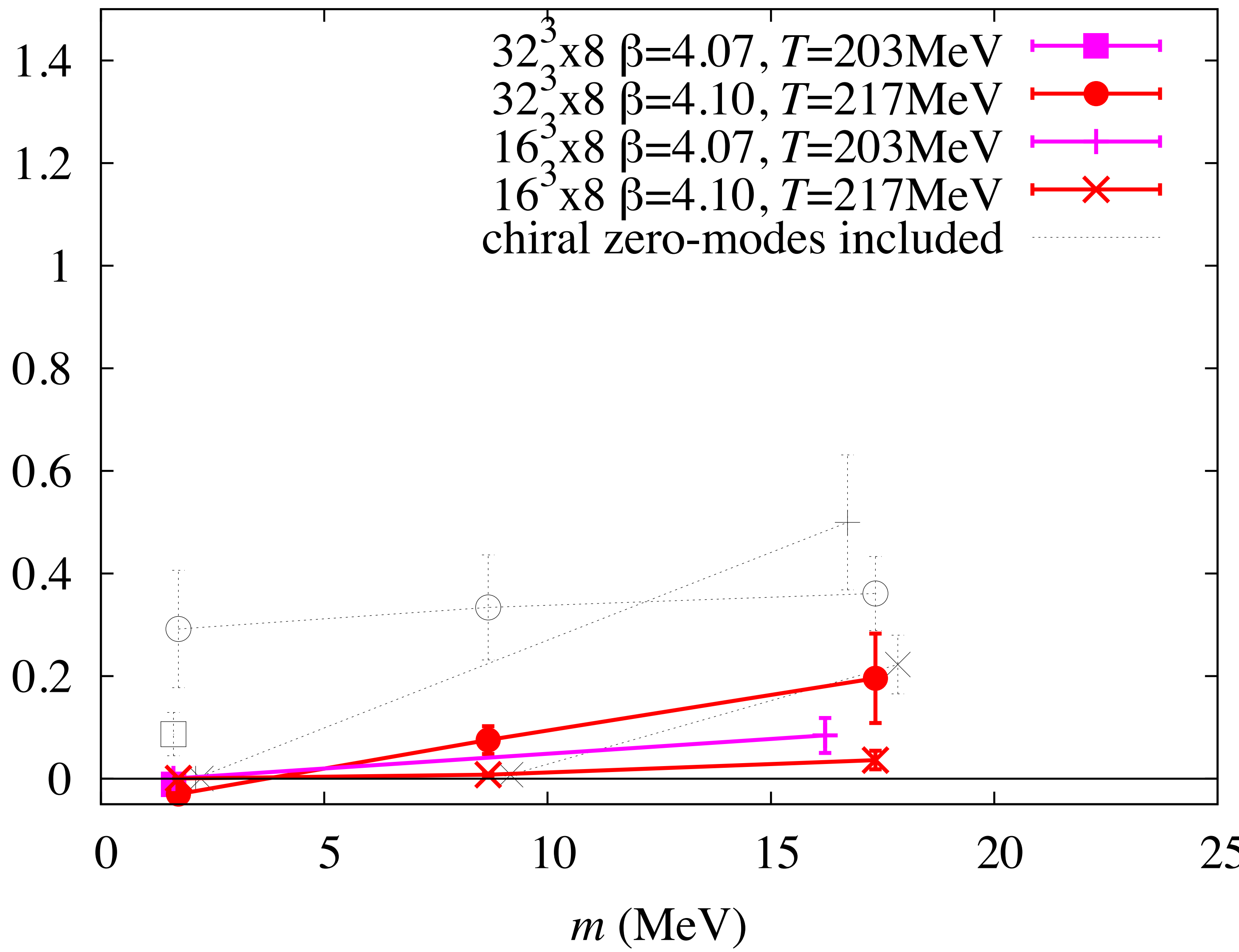
Even for finer lattice, ~40% are artifact

 arXiv: 1612.01908 & PRD 93, no. 3, 034507 (2016)

5. “GW violation” for DW fermion in low-laying modes

At the massless limit, overlap fermion suggests effective restoration of U(1)

$\chi_{U(1)_A}$
using OV eigenmodes



finer lattice as well

For overlap fermion, after taking of massless limit,
<u>physical U(1) violating signal is disappeared</u>

 arXiv: 1612.01908 & PRD 93, no. 3, 034507 (2016)

6. Summary

In this work, we examined axial U(1) breaking with Möbius domain-wall (DW), partially quenched overlap (on DW sea), and reweighted overlap fermions.

We found,

1. unexpectedly large violation of the Ginsparg-Wilson relation in low-laying modes of DW operator even for small residual mass case
2. precise **chiral symmetry both in sea and valence quark is crucial.**
3. reweighted overlap Dirac spectrum and susceptibility suggest $U(1)_A$ effective restoration at the chiral limit.

No more slides

Backups

Sym. of QCD<=>Degeneracy

$$\begin{tikzcd}
\langle \pi(x)\pi(0)\rangle \arrow[r, leftrightarrow, "SU(2)_L \times SU(2)_R"] \arrow[d, leftrightarrow, "U(1)_A"'] & \langle \sigma(x)\sigma(0)\rangle \arrow[d, leftrightarrow, "U(1)_A"] \\
\langle \delta(x)\delta(0)\rangle \arrow[r, leftrightarrow, "SU(2)_L \times SU(2)_R"] & \langle \eta(x)\eta(0)\rangle
\end{tikzcd}$$

$$\pi(x) = i\bar{\psi}(x)\gamma_5\tau\psi(x) \qquad \sigma(x) = \bar{\psi}(x)\psi(x)$$
$$\delta(x) = \bar{\psi}(x)\tau\psi(x) \qquad \eta(x) = i\bar{\psi}(x)\gamma_5\psi(x)$$

Degeneracy of these channels

<=> There are symmetries

First bin of ρ for the overlap

$L^3 \times L_t$	β	m	$\boxed{\rho_{\rm ov}(0\text{–}8\mathrm{MeV})}$	$\Delta^{\rm direct}_{\pi-\delta}a^2$	$\Delta^{\rm ev}_{\pi-\delta}a^2$	$\Delta^{\cancel{\rm GW}}_{\pi-\delta}/\Delta^{\rm ev}_{\pi-\delta}$	$\Delta^{\rm ov}_{\pi-\delta}a^2$	$\bar{\Delta}^{\rm ov}_{\pi-\delta}a^2$
$16^3 \times 8$	4.07	0.01	0.0071(18)	0.132(14)	0.139(12)	0.37(2)	0.19(5)	0.032(13)
$16^3 \times 8$	4.07	0.001	$3(3)\times10^{-12}$	0.032(7)	0.0498(14)	0.982(2)	0.00015(5)	$1.5(6)\times10^{-4}$
$16^3 \times 8$	4.10	0.01	0.0042(15)	0.073(12)	0.064(11)	0.278(40)	0.074(19)	0.012(6)
$16^3 \times 8$	4.10	0.005*	0.0008(3)	0.009(2)	–	–	0.0003(1)	0.003(1)
$16^3 \times 8$	4.10	0.001	$1.5(1.5)\times10^{-8}$	0.017(8)	0.0232(13)	0.983(4)	$6(3)\times10^{-5}$	$6(3)\times10^{-5}$
$32^3 \times 8$	4.07	0.001	0.00002(1)	0.105(32)	0.105(35)	0.65(10)	0.03(2)	-0.004(3)
$32^3 \times 8$	4.10	0.01	0.0067(14)	0.076(5)	0.069(5)	0.30(2)	0.120(24)	0.065(29)
$32^3 \times 8$	4.10	0.005	0.00147(20)	0.111(16)	0.107(15)	0.17(2)	0.111(34)	0.025(9)
$32^3 \times 8$	4.10	0.001	$1.5(1.3)\times10^{-5}$	0.036(11)	0.0125(50)	0.975(3)	0.097(38)	-0.010(5)
$32^3 \times 12$	4.23	0.01	0.011(1)	0.112(10)	0.109(4)	0.038(4)	0.11(1)	0.064(11)
$32^3 \times 12$	4.23	0.005	0.00444 (96)	0.107(11)	0.107(8)	0.083(9)	0.115(16)	0.026(7)
$32^3 \times 12$	4.23	0.0025	0.0017(4)	0.186(47)	0.216(41)	0.162(22)	0.162(40)	0.0065(20)
$32^3 \times 12$	4.24	0.01	0.011(1)	0.135(8)	0.101(3)	0.046(3)	0.107(14)	0.065(10)
$32^3 \times 12$	4.24	0.005	0.0054(9)	0.112(17)	0.124(13)	0.057(10)	0.122(21)	0.030(14)
$32^3 \times 12$	4.24	0.0025	0.0008(5)	0.052(15)	0.041(13)	0.32(8)	0.078(52)	0.0030(6)

(skip)

Re-weighting tech. enables us to change another fermion determinant (= quark loop effect exchange)

$$
\begin{aligned}
\langle \mathcal{O} \rangle_{\text{Overlap}} &\propto \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu \, \mathcal{O} \, e^{-S_{\text{gauge}}} e^{-\bar{\psi}[D_{\text{OV}}]\psi} \\
&= \int \mathcal{D}A_\mu \, \mathcal{O} \, e^{-S_{\text{gauge}}} \mathrm{Det}[D^2_{\text{OV}}] \\
&= \int \mathcal{D}A_\mu \, \mathcal{O} \, e^{-S_{\text{gauge}}} \mathrm{Det}[D^2_{\text{OV}}] \frac{\mathrm{Det}[D^2_{\text{DW}}]}{\mathrm{Det}[D^2_{\text{DW}}]} \\
&= \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu \, \mathcal{O} R \, e^{-S_{\text{gauge}}} e^{-\bar{\psi}[D_{\text{DW}}]\psi} \\
&\propto \langle \mathcal{O} R \rangle_{\text{Domain Wall}}
\end{aligned}
$$

$$
R = \frac{\mathrm{Det}[D^2_{\text{OV}}]}{\mathrm{Det}[D^2_{\text{DW}}]}
$$

Multiplying R and taking average, we obtain the result with the overlap determinant

$$m_{\mathrm{res}} = \frac{\langle \mathrm{tr}\, G^\dagger \Delta_L G \rangle}{\langle \mathrm{tr}\, G^\dagger G \rangle}, \quad \Delta_L = \frac{1}{2}\gamma_5(\gamma_5 D_{\mathrm{DW}}^{4D} + D_{\mathrm{DW}}^{4D}\gamma_5 - 2aD_{\mathrm{DW}}^{4D}\gamma_5 D_{\mathrm{DW}}^{4D}),$$

G: contact-term-subtracted quark propagator,

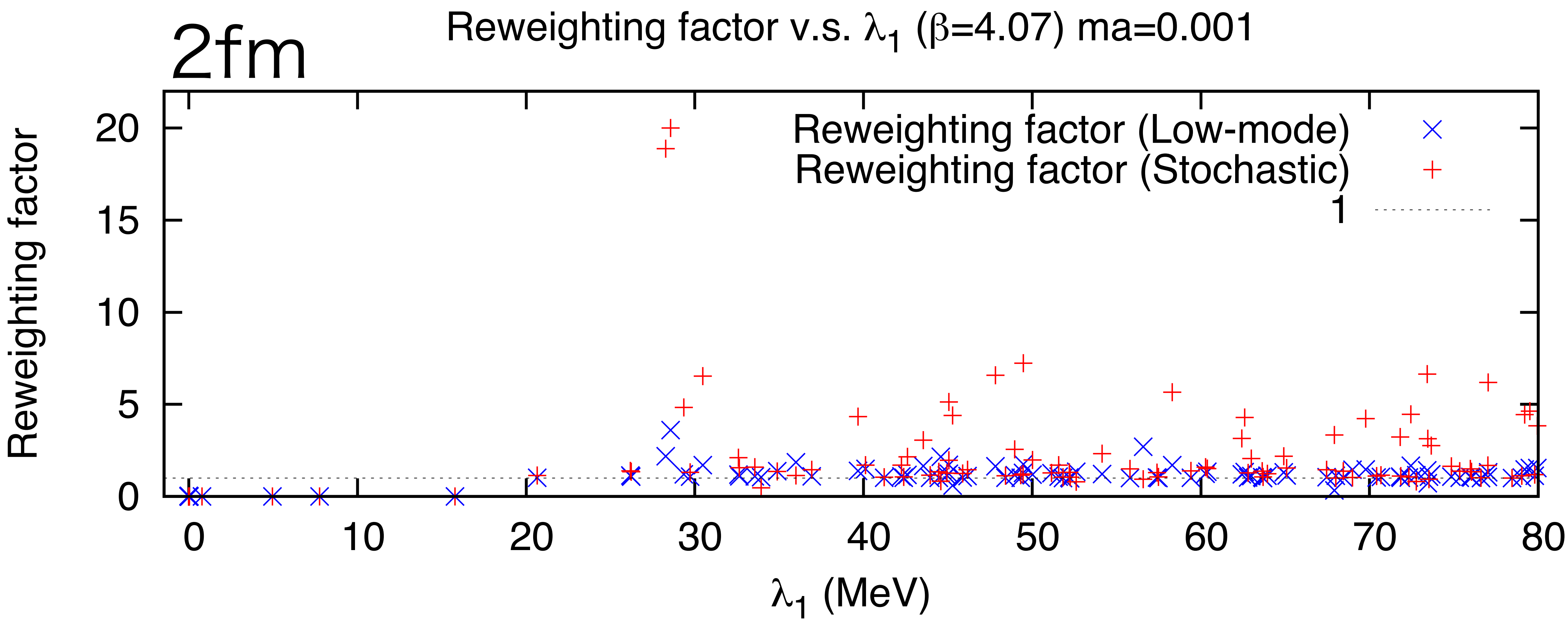
R with UV suppression factor
low-mode reweighting

$$R(A) = \frac{\mathrm{Det}\, D^2_{\mathrm{ov}}(m)}{\mathrm{Det}\, D^2_{\mathrm{DW}}(m)} \frac{\mathrm{Det}\, D^2_{\mathrm{DW}}(1/2a)}{\mathrm{Det}\, D^2_{\mathrm{ov}}(1/2a)}. \quad (\text{for } L = 16^3 \times 8)$$

$$R(A) \sim \frac{\prod_i^{N_{th}} [({\lambda_{\mathrm{ov}}}^i_m)^2]}{\prod_i^{N_{th}} [({\lambda_{\mathrm{DW}}}^i_m)^2]} = R_{\mathrm{low}}(A), \quad (\text{for } L = 16^3 \times 8,\ 32^3 \times 8)$$

Low-mode reweighting factor does not seems to affect existence of the gap

2fm



This is now testing in finer (and larger) lattice...

Massless Dirac spectrum

The Dirac spectrum of the massless fermion can be obtained by subtracting,

$$\lambda_i a \equiv \frac{\sqrt{a^2(\lambda_i^m)^2 - a^2 m_{\mathrm{ud}}^2}}{\sqrt{1 - a^2 m_{\mathrm{ud}}^2}},$$

We measure the violation of the Ginsparg-Wilson relation on each eigenmode of the Hermitian Dirac operator $\gamma_5 D$ through

$$g_i \equiv \frac{\psi_i^\dagger \gamma_5[D\gamma_5 + \gamma_5 D - 2aD\gamma_5 D]\psi_i}{\lambda_i^m} \left[\frac{(1-am_{\mathrm{ud}})^2}{2(1+am_{\mathrm{ud}})}\right], \tag{7.2}$$

where λ_i^m, ψ_i denotes the i–th eigenvalue/eigenvector of massive hermitian Dirac operator respectively. D is the domain-wall or overlap Dirac operator. Last factor in (7.2) comes from the normalization of the Dirac operator. Note that one can obtain the residual mass by an weighted average of g_i,

$$m_{\mathrm{res}} = \frac{\langle \mathrm{tr}\, G^\dagger \Delta_L G\rangle}{\langle \mathrm{tr}\, G^\dagger G\rangle} = \sum_i \frac{\lambda_i^m(1+am_{\mathrm{ud}})}{(1-am_{\mathrm{ud}})^2(a\lambda_i^m)^2} g_i \Bigg/ \sum_i \frac{1}{(a\lambda_i^m)^2}. \tag{7.3}$$

where the sum runs over all eigenvalues.

Reweighting factor

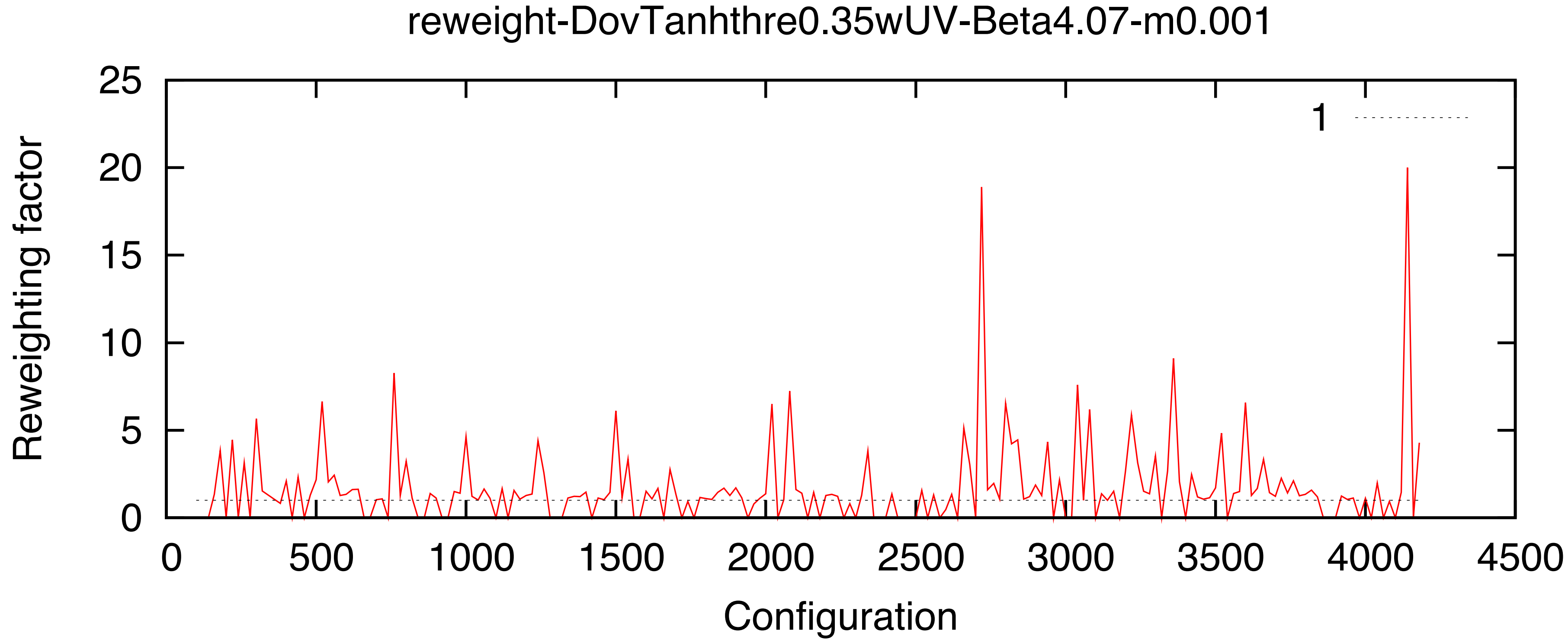
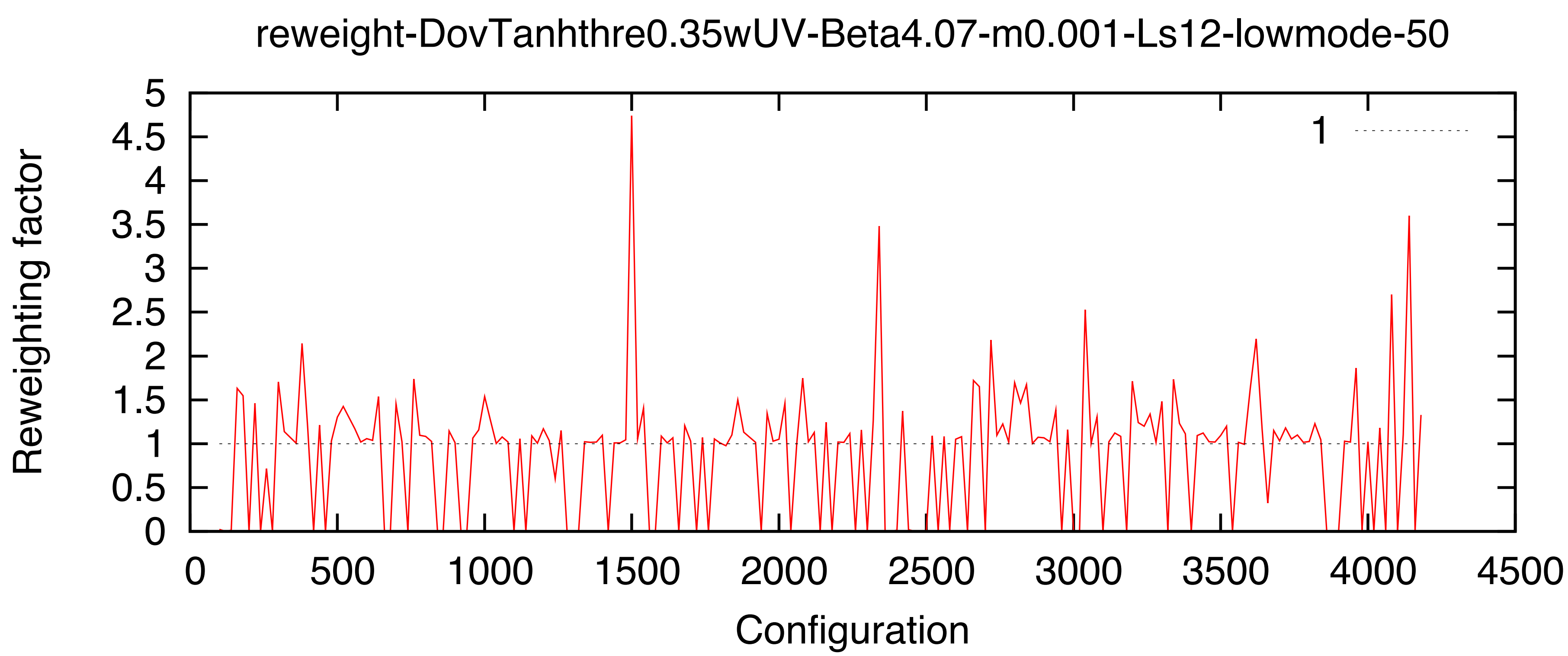
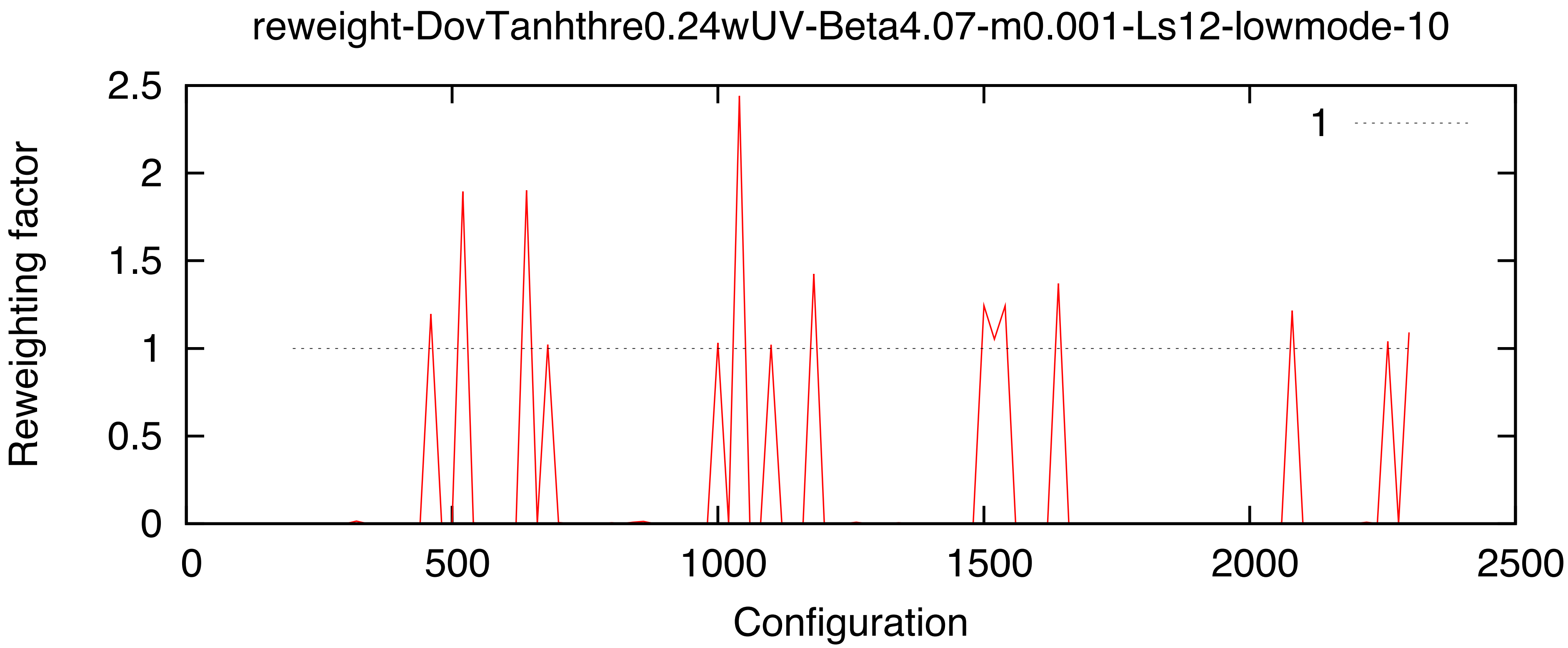


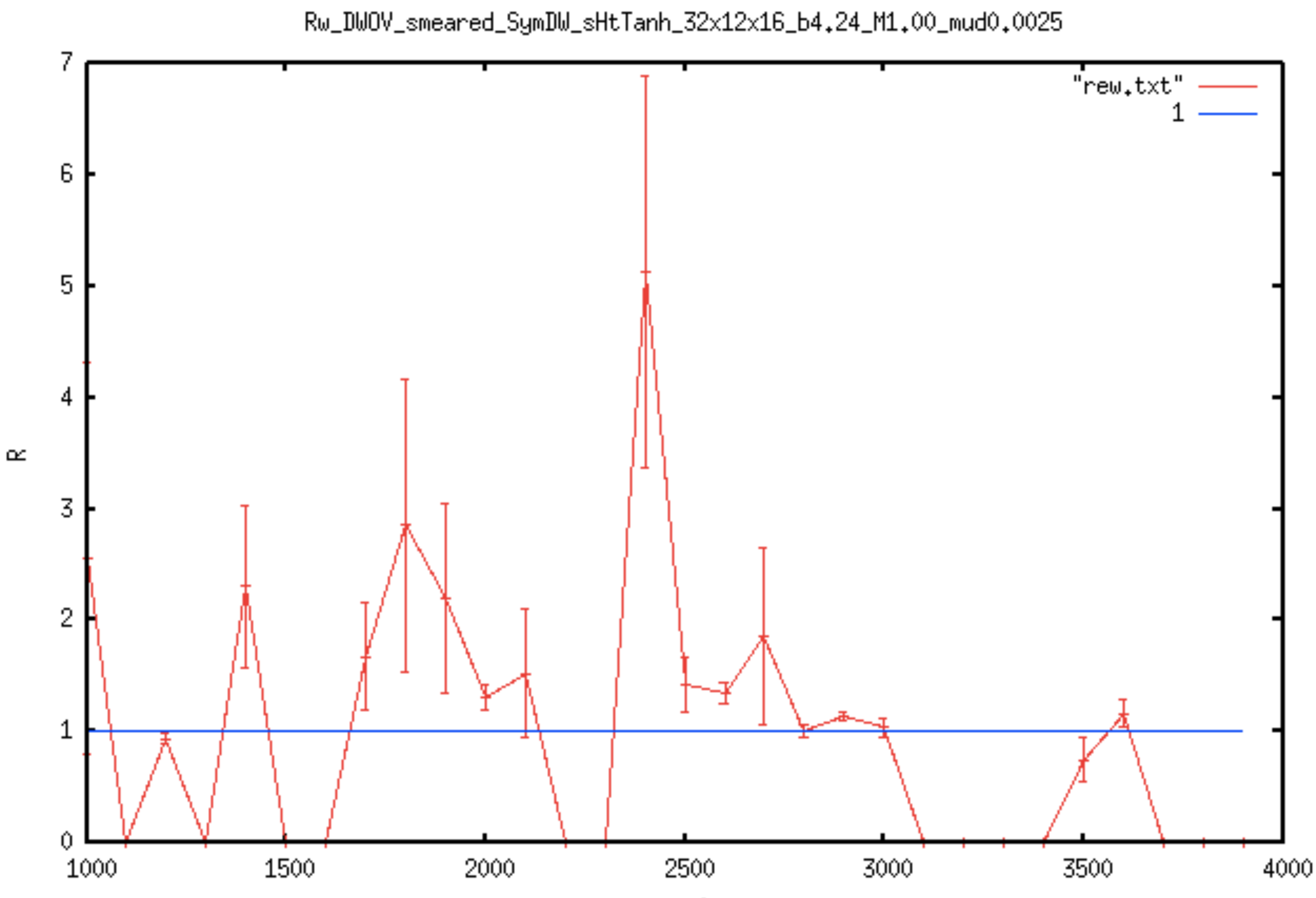
FIG. 30:





reweight-DovTanhthre0.24wUV-Beta4.07-m0.001-Ls12-lowmode-10
Reweighting factor
Configuration
1
0
0.5
1
1.5
2
2.5
0
500
1000
1500
2000
2500

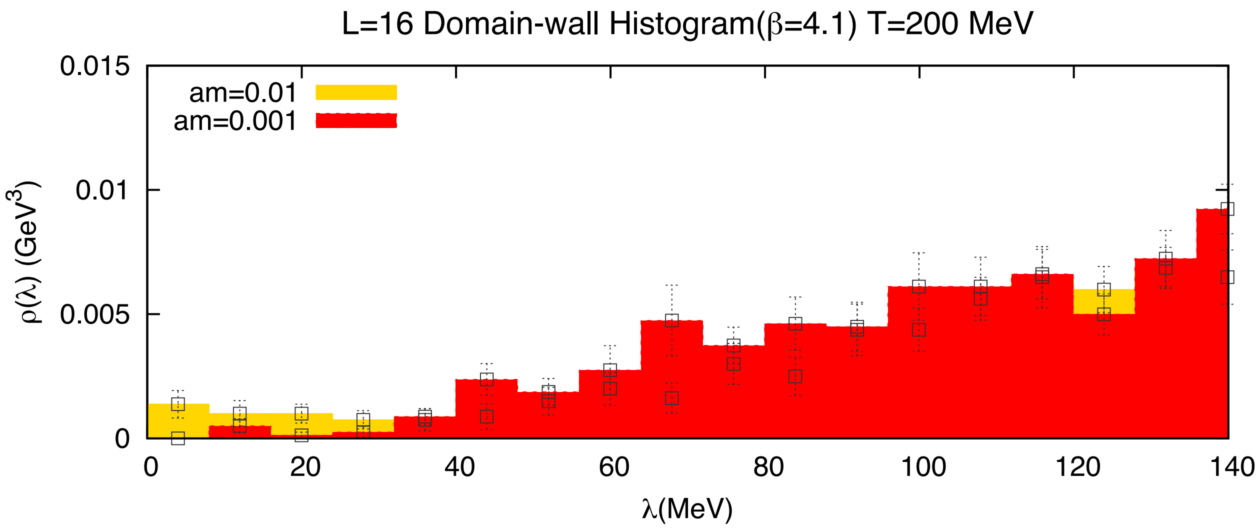
Reweighting factors vs configuration

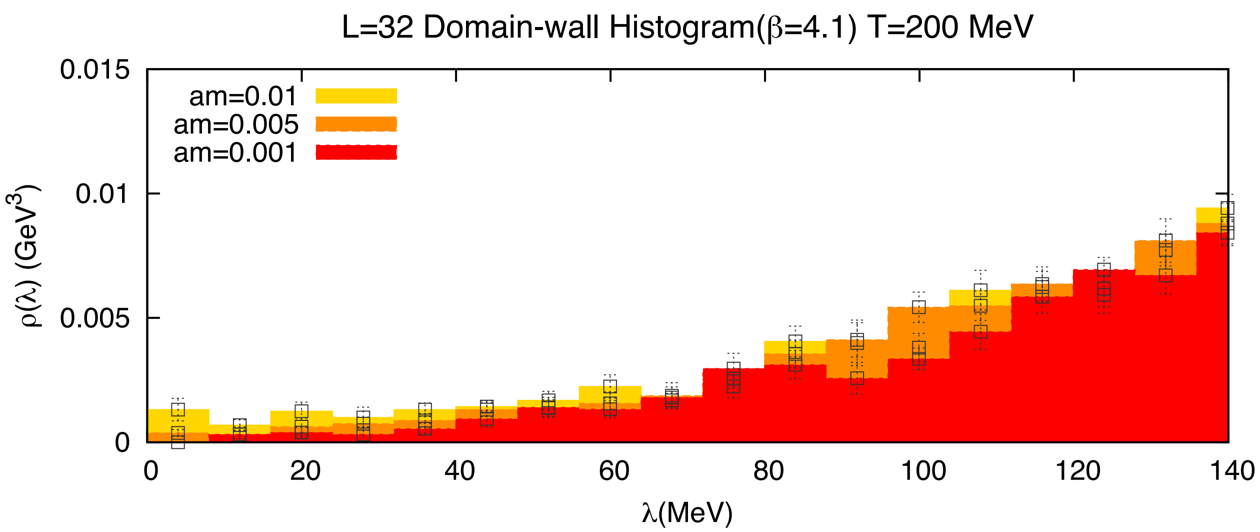


T>Tc

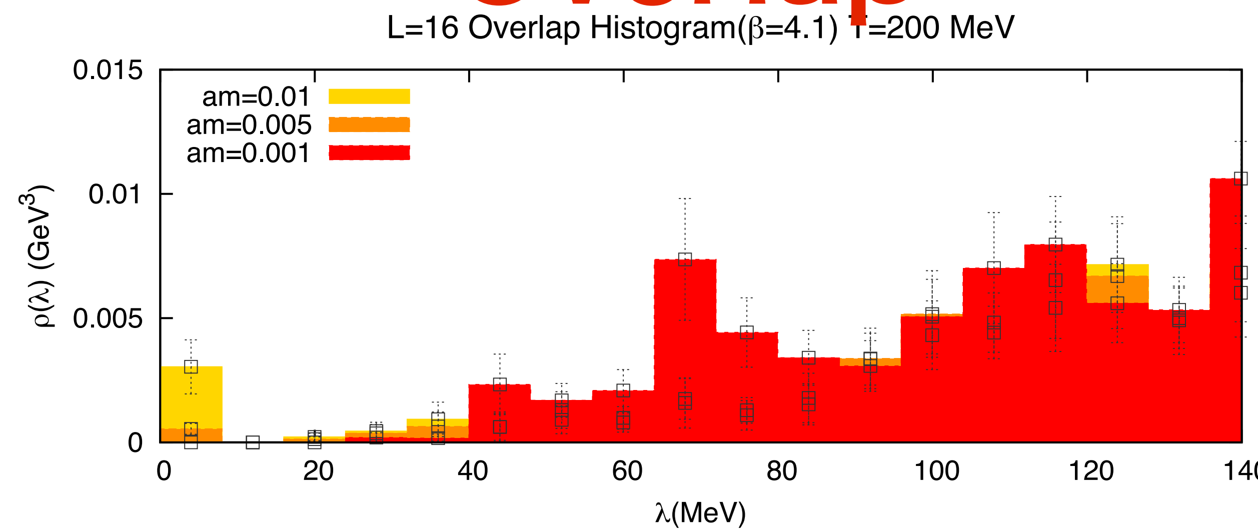
(skip)

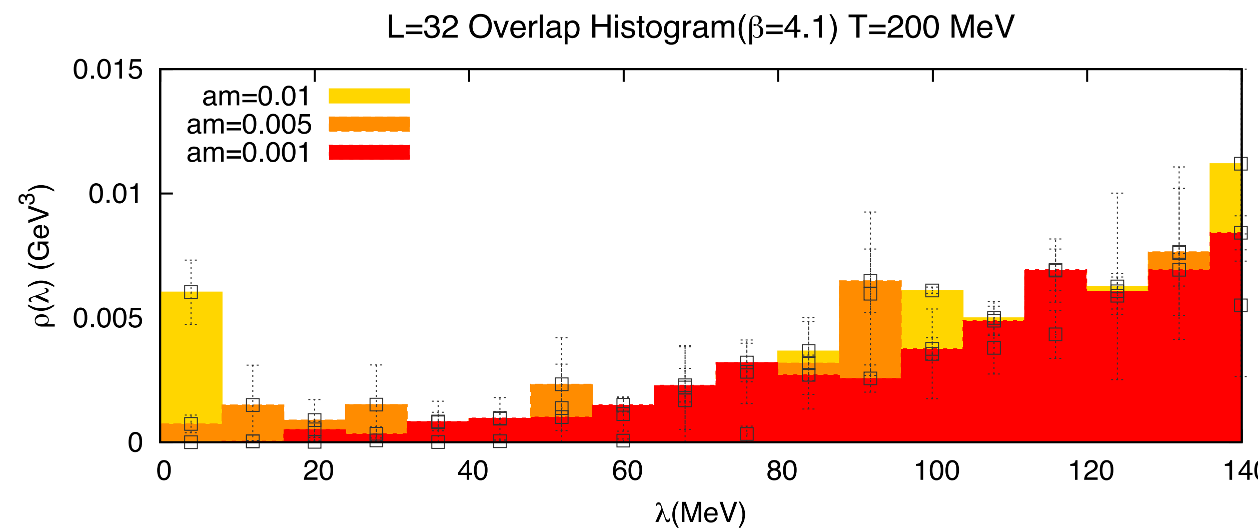
Domain-wall



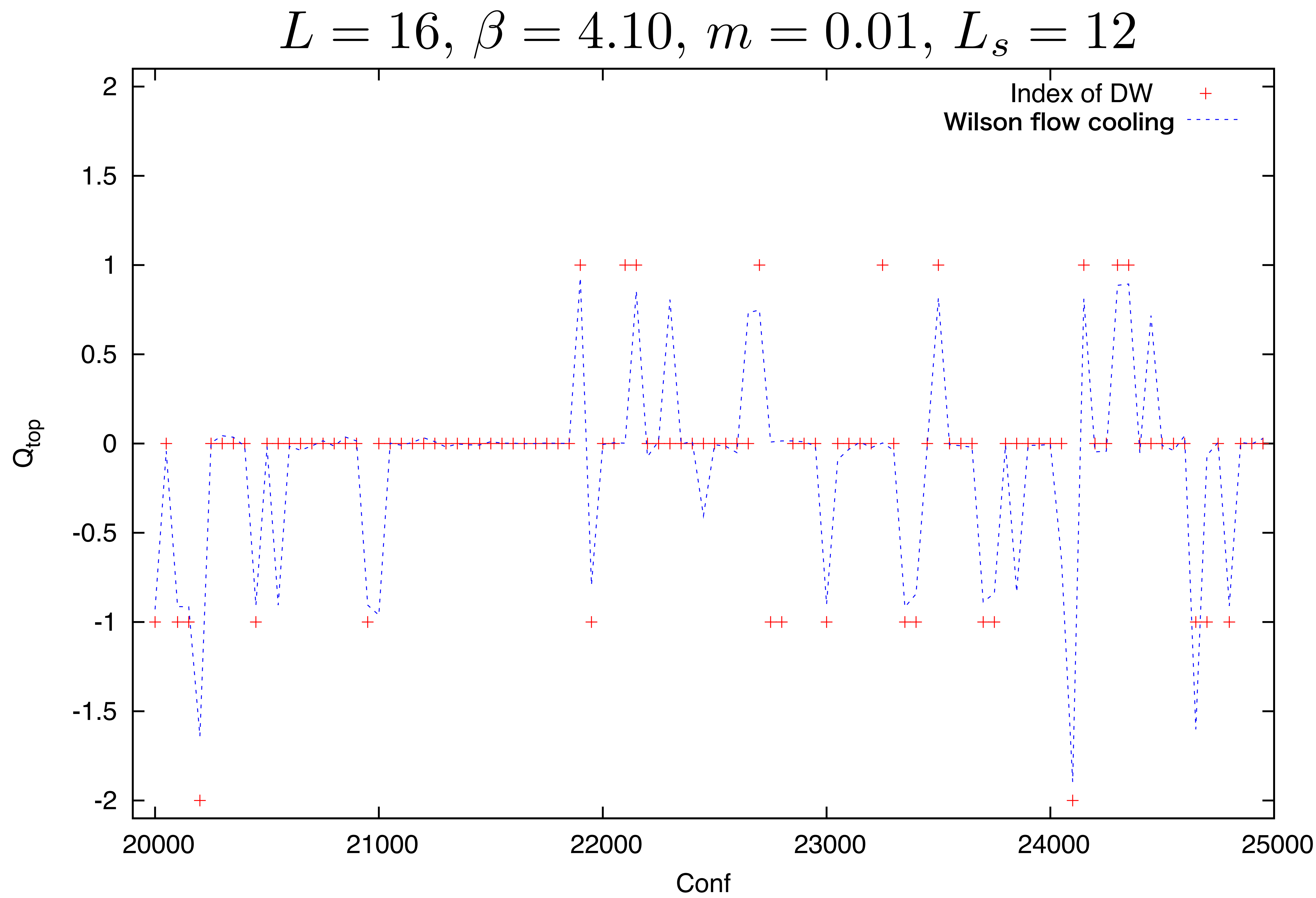


Overlap

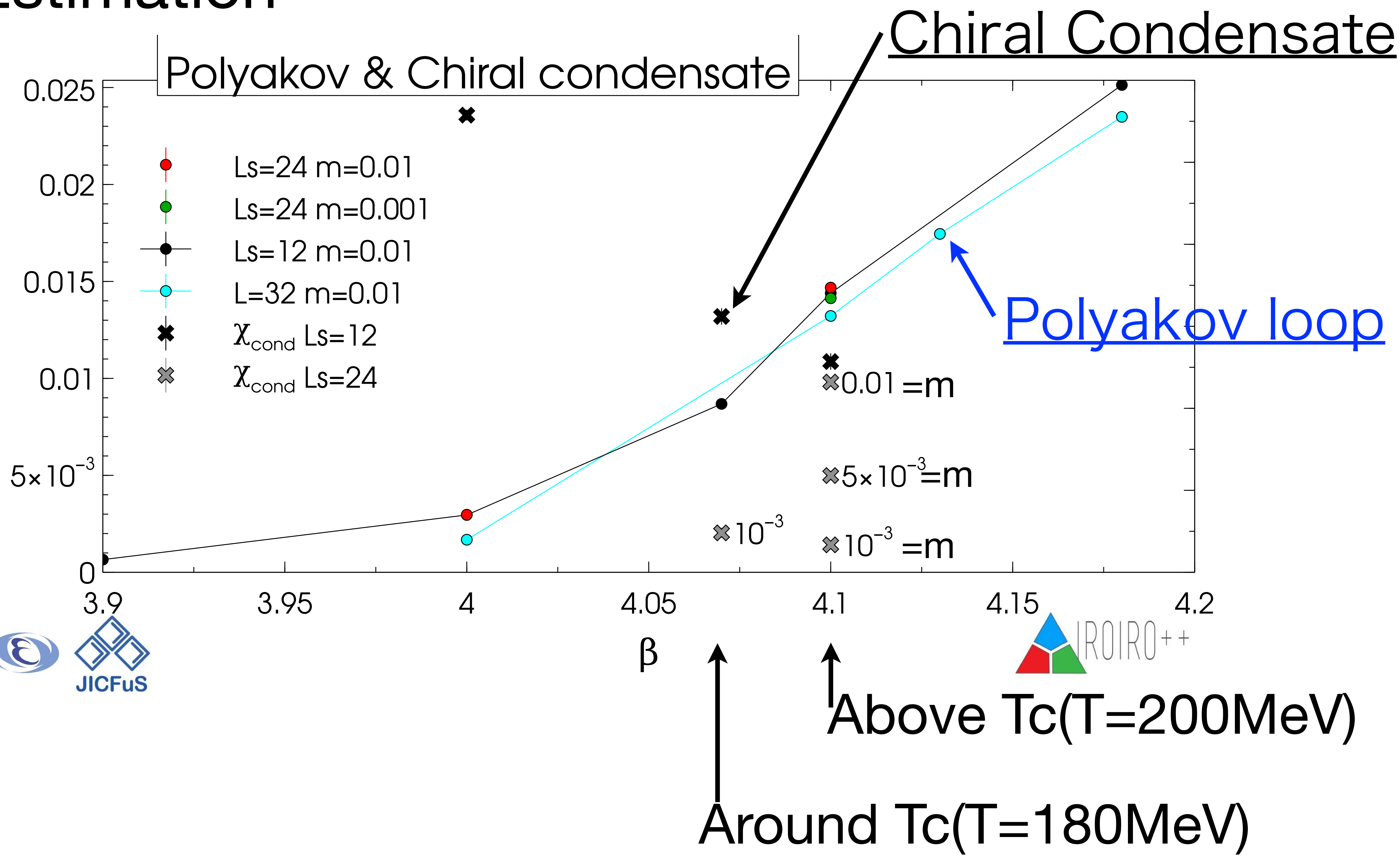




Topological charge changes along HMC



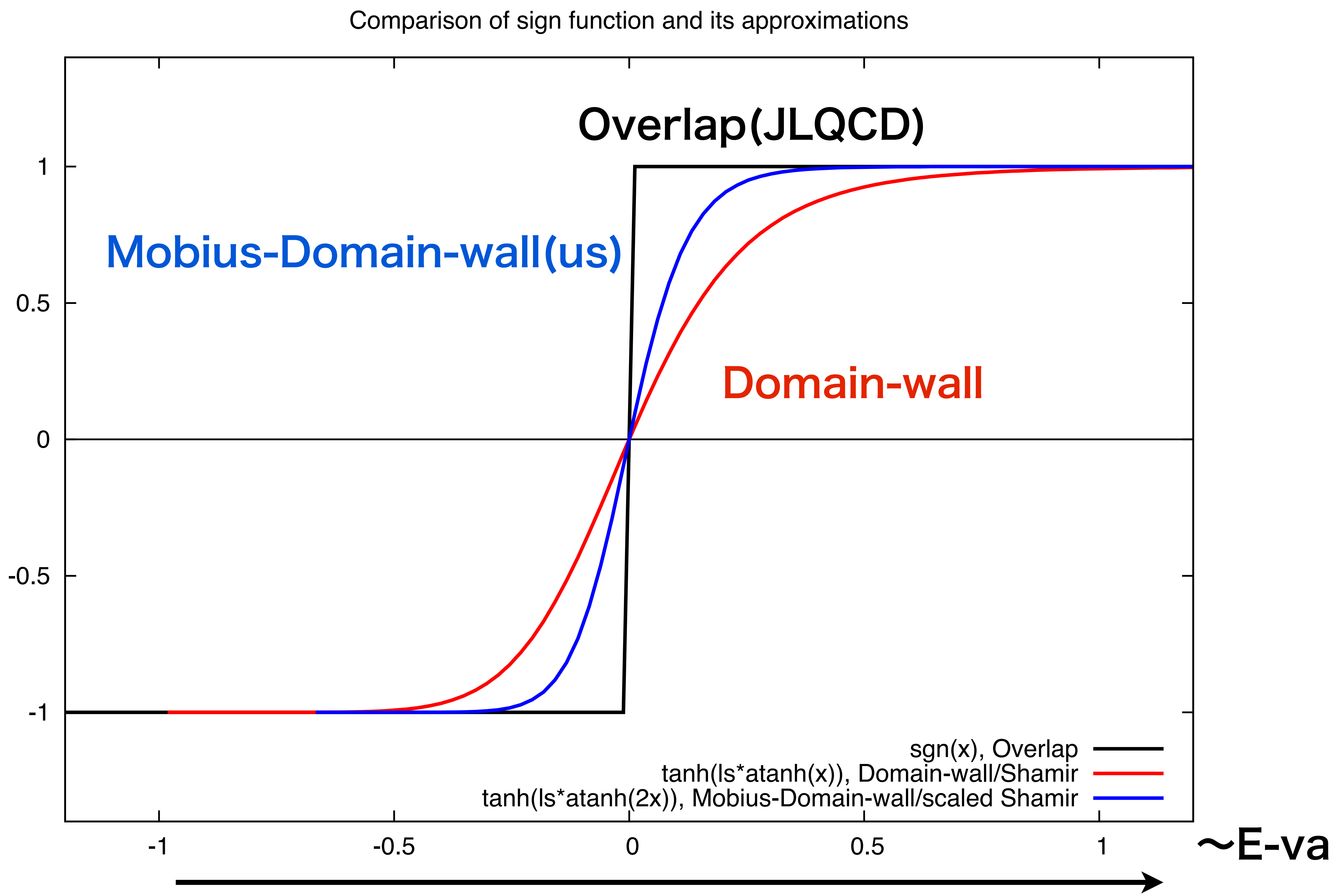
Tc Estimation



Vol. dependence of Polyakov loop

Decreasing of Chiral condensate

Overlap type=Different “Sign function”



$$\mathrm{U}(2)_{\mathrm{L}} \times \mathrm{U}(2)_{\mathrm{R}} \;\simeq\; \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}} \times \mathrm{U}(1)_V \times \mathrm{U}(1)_A \;, \tag{3.10}$$

where $\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}}$ symmetry corresponds to

$$\psi \to e^{i\theta\gamma_5\tau^a}\psi, \tag{3.11}$$

$$\bar{\psi} \to \bar{\psi}e^{+i\theta\tau^a\gamma_5}, \tag{3.12}$$

(the SU(2) chiral symmetry) and

$$\psi \to e^{i\theta\tau^a}\psi, \tag{3.13}$$

$$\bar{\psi} \to \bar{\psi}e^{-i\theta\tau^a}. \tag{3.14}$$

On the other hand, the $\mathrm{U}(1)_{\mathrm{A}}$ symmetry, equivalently the U(1) chiral symmetry, corresponds to

$$\psi \to e^{i\theta\gamma_5}\psi, \tag{3.15}$$

$$\bar{\psi} \to \bar{\psi}e^{+i\theta\gamma_5}. \tag{3.16}$$

Cohen's argument :

$$\Pi_\sigma(x) - \Pi_\delta(x) = \frac{1}{Z}\int [\mathcal{D}A] e^{-S_{\mathrm{YM}}} \mathrm{Det}\left[\not{D} - m\right]\left[\mathrm{Tr}\left[G(x,x)\right]\mathrm{Tr}\left[G(0,0)\right]\right]$$

$$\langle\bar{\psi}\psi\rangle = \frac{1}{Z}\int [\mathcal{D}A] e^{-S_{\mathrm{YM}}} \mathrm{Det}\left[\not{D} - m\right]\mathrm{Tr}\left[G(x,x)\right]$$

$$\begin{aligned}\mathrm{Tr}\left[G(x,x)\right] &= \sum_j \frac{-m\psi_j^\dagger(x)\psi_j(x)}{\lambda_j^2 + m^2} \\ &= \int d\lambda \frac{-m\rho_A(\lambda)}{\lambda^2 + m^2}\end{aligned}$$

$$e^{-S_{\mathrm{YM}}} \mathrm{Det}\left[\not{D} - m\right]\mathrm{tr}\left[G(x,x)\right] = 0,$$